Fundamentals of Nuclear Magnetic Resonance Absorption. II*†

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(Received April 6, 1950)

CONTENTS OF PART II

E. Experimental

13. Apparatus for Detecting Nuclear Resonance Absorption

Gorter[23] in 1936 attempted unsuccessfully to detect heating of the lattice as a consequence of the energy absorbed by the spin system from the r-f field and transferred to the lattice through the relaxation mechanism. Later, Gorter and Broer[24] failed in an effort to observe nuclear paramagnetic dispersion.

The first successful absorption experiment was carried out by Purcell, Torrey, and Pound and reported in January, 1946.[4] A resonant cavity, filled with about a pound of paraffin, was excited at 30 Mc sec^{-1} in a mode characterized by an oscillatory magnetic field circulating around the center post of the cavity. With the external magnetic field parallel to the center post, the cavity was placed in one half of the divided circuit shown in Fig. 5. The attenuator and phase shifters were adjusted to produce a null signal at a value of H_0 off resonance for the applied generator frequency. As the external field H_0 was slowly varied through the resonance condition for protons (known from molecular beam experiments) a change in the transmitted r-f power was observed.
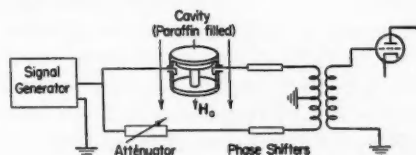

FIG. 5. Schematic diagram of the apparatus used in the first successful resonance absorption experiment.

²³ C. J. Gorter, *Physica* 3, 995 (1936).
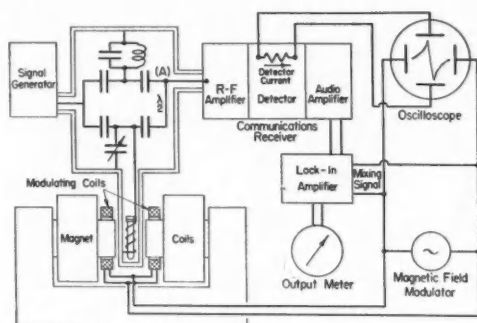²⁴ C. J. Gorter and L. J. F. Broer, *Physica* 9, 591 (1942).

FIG. 6. Schematic diagram of the modulation bridge used by Bloembergen, Purcell and Pound.

A disadvantage of the cavity arrangement is that sensitivity cannot be increased by increasing the power level. For this reason, Purcell's group turned to a bridge[5] using lumped circuit elements, pictured schematically in Fig. 6. This is only one of several possible bridge circuits, the essential feature of which is the balancing out of signal generator fluctuations; the extra half-wave of cable in one arm of the bridge provides a voltage node at point A when balance is achieved. The sample occupies a volume the order of 1 cm^3 within the r-f coil of a tuned circuit, the axis of the coil being perpendicular to the external magnetic field H_0. A similar tuned circuit with no sample forms the other arm of the bridge. A preamplifier may be employed to obtain an optimum ratio of the measured signal to the spurious noise voltages arising in the various circuit components.

The half-wave of cable, which is several meters long for frequencies usually encountered in nuclear resonance, introduces some inconvenience as well as occasional instability. An accidental tug at the cable or an apparently insignificant change in its position may sufficiently alter its effective length to throw the bridge off balance, and bridges which eliminate it often exhibit greater stability. Such bridges may involve an r-f transformer which incorporates balance through different winding directions for each half of the primary, or they may achieve balance through use of a nonresonant T-section[25] in the branch of the bridge which previously contained the dummy resonant circuit and half-wave of cable.

[25] H. L. Anderson, *Physical Rev.* **76**, 1460 (1949).

The procedure for examining the resonance involves first balancing the bridge when the nuclear system is out of resonance. Then the external magnetic field is adjusted to meet the resonant condition for the fixed radiofrequency $\omega/2\pi$ at which the signal generator excites the bridge. It is convenient to modulate H_0 by superimposing on it a small amplitude low frequency magnetic field parallel to it. Thus, when H_0 is set near resonance, the nuclear Larmor frequency varies in and out of resonance, and resonance absorption may occur in the sample arm of the bridge. The resulting unbalance signal is amplified and detected by the receiver, and may be placed on the vertical plates of an oscilloscope. In the following section it is seen that the oscilloscope pictures either χ'' or χ' if the bridge is properly balanced.

14. Relation of the Bridge Signal to the Nuclear Susceptibilities

Consider the tuned circuit containing the coil in which the sample resides (Fig. 7). The inductance $\mathcal{L}$ of the coil is proportional to the nuclear permeability which depends upon the nuclear susceptibility:

$$\mathcal{L} = L_0(1+4\pi\chi). \qquad (14.01)$$

We assume the circuit to be tuned to circuit resonance ($\omega C = 1/\omega L_0$) when the bridge is balanced off resonance, so that the input admittance at or near nuclear resonance

$$\mathcal{Y} = \frac{1}{R} + i\left(\omega C - \frac{1}{\omega L_0(1+4\pi\chi)}\right) \quad (14.02)$$

becomes

$$\mathcal{Y} = \frac{1}{R} + i\frac{4\pi\chi}{\omega L_0} \qquad (14.03)$$

if one makes use of $\omega C = 1/\omega L_0$ and the fact that $|4\pi\chi|$ is very much less than unity. It is instructive to note that the imaginary portion of χ leads to a real or dissipative term in $\mathcal{Y}$, in
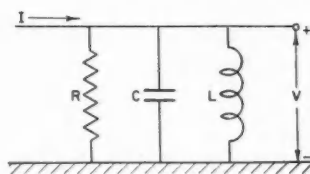

FIG. 7. Circuit considered in the analysis of a branch of the bridge.

agreement with our previous finding that absorption is proportional to χ''.

When the circuit is supplied by a constant current source $\mathcal{I}$, the potential drop across it is

$$\mathcal{V} = \mathcal{I}/\mathcal{Y} = \mathcal{I}R\left[1 + i\frac{4\pi\chi R}{\omega L_0}\right]^{-1}. \quad (14.04)$$

The circuit Q is $R/\omega L_0$, and, writing $\mathcal{I}R = \mathcal{V}_0$, we have

$$\mathcal{V} \cong \mathcal{V}_0[1 - i4\pi Q\chi], \quad (14.05)$$

where it has been assumed that $|4\pi Q\chi| \ll 1$.

The voltage $\mathcal{V}$ is compared at point A of the bridge with an out-of-phase signal of nearly equal magnitude from the opposite arm of the bridge. If this is denoted $-\mathcal{V}_1$, the signal fed into the receiver is proportional to

$$v = \mathcal{V} - \mathcal{V}_1 = \mathcal{V}_0 - \mathcal{V}_1 - 4\pi Q\mathcal{V}_0(\chi'' + i\chi') \quad (14.06)$$

and we may think of the two vectors $\mathcal{V}_0 - \mathcal{V}_1$ and

$$v_x = -4\pi Q\mathcal{V}_0(\chi'' + i\chi') \quad (14.07)$$

as combining to give v. Upon letting θ be the phase between $\mathcal{V}_0$ and $\mathcal{V}_0 - \mathcal{V}_1$, we obtain from Eq. (14.06)

$$v = \mathcal{V}_0 - \mathcal{V}_1 - 4\pi Q|\mathcal{V}_0|e^{i\theta}(\chi'' + i\chi'), \quad (14.08)$$

if $\mathcal{V}_0 - \mathcal{V}_1$ is taken for the present to be real. Ordinarily $|v_x|$ is appreciably smaller than $\mathcal{V}_0 - \mathcal{V}_1$ and the magnitude of v is given to good approximation by adding to $\mathcal{V}_0 - \mathcal{V}_1$ the component of v_x along $\mathcal{V}_0 - \mathcal{V}_1$. In Eq. (14.08), therefore, $|v|$ is simply the real part, and

$$|v| = \mathcal{V}_0 - \mathcal{V}_1 \\ + 4\pi Q|\mathcal{V}_0|(-\chi''\cos\theta + \chi'\sin\theta). \quad (14.09)$$

As the oscilloscope trace sweeps back and forth it plots above the base line a function proportional to

$$4\pi Q|\mathcal{V}_0|(-\chi''\cos\theta + \chi'\sin\theta), \quad (14.10)$$

and two cases are readily distinguishable:

Case I. $\theta = 0$ or π.—The oscilloscope plots the *absorption curve* χ'' under this condition of *amplitude unbalance*, since $\mathcal{V}_0 - \mathcal{V}_1$ must be along $\mathcal{V}_0$ and the residual unbalance represents an amplitude difference between the two signals arriving at point A in the bridge.

Case II. $\theta = \pi/2$ or $3\pi/2$.—The oscilloscope plots the *dispersion curve* χ' under this condition
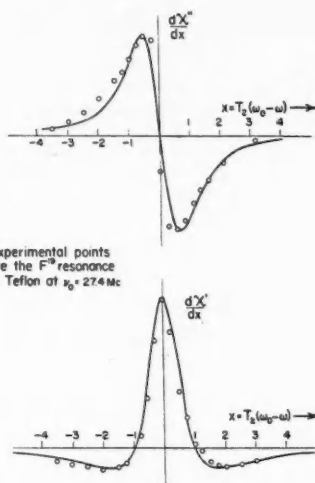

FIG. 8. Experimental points for the fluorine resonance in Teflon plotted over curves representing the derivatives of the Bloch susceptibilities. The value of T_2 is taken to be 2.27×10^{-5} sec in making this comparison.

of *phase unbalance* since $\mathcal{V}_0 - \mathcal{V}_1$ and $\mathcal{V}_0$ have the same amplitude but differ slightly in phase on arriving at point A.

Intermediate values of θ lead to unwelcome mixtures of χ' and χ'' which are readily recognized by their lack of symmetry properties exhibited in Fig. 4. From the foregoing argument it is also evident that, to obtain a faithful reproduction of χ' or χ'', it is essential that the bridge is *not* balanced so well that $\mathcal{V}_0 - \mathcal{V}_1$ becomes of comparable magnitude to v_x, for then Eq. (14.09) is not valid.

Using representative values of $Q = 100$, $|\mathcal{V}_0| = 10^{-1}$ volt, and taking into account that resonant nuclear susceptibilities vary from less than 10^{-7} to about 10^{-5}, one sees that v_x ranges from less than 10 microvolts to a maximum of perhaps a millivolt. Thus thermal noise and shot noise effects may offer severe competition to the nuclear signal we wish to detect, and a narrow band "lock-in" amplifier is used in increasing sensitivity. This device may be thought of as multiplying the audio signal from the receiver by a sinusoid of the modulation frequency and measuring the time average of the resultant. The mathematical orthogonality of sines and cosines leads to zero for this average unless the signal has a component of proper phase at the modulation fre-
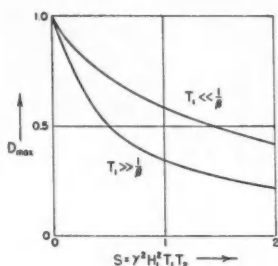
FIG. 9. Variation of the maximum meter deflection with the saturation factor; β is the modulation frequency.

quency. The signal-to-noise ratio is increased by the reduction of the noise accepted by the measuring device to a narrow portion of the noise spectrum centered about the modulation frequency.

A significant feature in using a "lock-in" amplifier is the reduction of modulation amplitude to a fraction of the line-width expressed in gauss, whereas one would use several line widths in order to display χ' or χ'' on the oscilloscope. If the ratio a of the modulation amplitude to the line width $(\gamma T_2)^{-1}$ is less than unity, it can be shown by means of a Taylor's expansion that the lock-in meter deflection D is proportional to $d\chi'/dH_0$ or to $d\chi''/dH_0$ as H_0 varies through the resonance:

$$D \sim 4\pi Q |\mathcal{U}_0| a\left[\frac{d}{dx}(-\chi'' \cos\theta + \chi' \sin\theta)\right], \quad (14.11)$$

where $x = T_2(\omega_0 - \omega)$. The derivatives of the Bloch susceptibilities are shown in Fig. 8 along with experimental points for the F^{19} resonance in polytetrafluoroethylene (Teflon), a substance for which the Bloch theory seems quite adequate.

When a approaches unity, small amounts of the higher odd-order derivatives are admixed. However, even $a=1$ introduces but $\frac{1}{8}$ part of the third derivative and 1/192 part of the fifth derivative, and a need not be very much smaller than unity in order for a reasonably faithful first derivative to be measured.

A bridge circuit, of course, is not the only experimental arrangement capable of observing the dependence of the sample-coil inductance upon the resonant susceptibilities. For example, the sample may be placed in the coil of the tank circuit of an oscillator, and the effect of the resonance on either the oscillation level or the oscillator frequency can be observed. A number

of "bridgeless" radiofrequency spectrometers have been used in nuclear resonance experiments.[26-29] Although these will not be discussed in detail here, it is evident that such an instrument should offer advantages in that varying the capacitance in the tank circuit permits systematic searching in frequency for new resonances in a fixed magnetic field. Pound has measured a number of nuclear magnetic moments in this way, and Knight[29] has discovered with his apparatus an interesting small shift in the resonance frequency in metals.

15. Transient Phenomena and the Direct Measurement of T_1

Inasmuch as T_1 and T_2 are characteristic times relating to fundamental nuclear spin processes, it is not surprising that these parameters are involved in describing transient responses of the nuclear spin system and that they may be determined by experimentally observing these responses. Such transients are expected to follow the general solution of the Bloch equations, and the recent more accurate methods[18,30] of measuring T_1 are based upon more general solution of the Bloch equations than was discussed in Sec. 11.

We shall discuss in this and the following sections two fundamental methods for measuring T_1, which happen to have been the two methods first used in relaxation studies. For discussions of the transient methods, the student is referred to the article by Torrey, and to the analysis of the "wiggles" phenomenon (BPP, p. 692) by Jacobsohn and Wangsness.[31]

The spin-lattice relaxation time was defined (Sec. 8) as the time for all but $1/e$ of the excess nuclei appropriate to a Boltzmann distribution to have reached the lower energy state $(I = \frac{1}{2})$. It is possible, for sufficiently long T_1, to observe this establishment directly and thereby to measure T_1. There are several variations of technique, depending upon the line width, the signal strength, and the value of T_1.

[26] R. V. Pound, *Physical Rev.* **72**, 527 (1947).
[27] A. Roberts, *Rev. Sci. Inst.* **18**, 845 (1947).
[28] J. R. Zimmerman and D. Williams, *Physical Rev.* **76**, 350 (1949).
[29] W. D. Knight, *Physical Rev.* **76**, 1259 (1949).
[30] H. C. Torrey, *Physical Rev.* **76**, 1059 (1949).
[31] B. Jacobsohn and R. Wangsness, *Physical Rev.* **73**, 942 (1948).

For liquid samples, one may set the r-f power at a sufficiently high level to produce saturation; the modulation amplitude is adjusted to be almost exactly equal to the line width. If, then, the modulation is suddenly increased ten or twentyfold, and if the resonance occurs on the oscilloscope relatively near the center of the trace, the time during which the spin system is in resonance, and therefore the total r-f energy delivered to the spin system per second, will be reduced by 1/10 or 1/20. As a consequence, the excess number will approach a larger value appropriate to a less saturated state, with the characteristic approach function $[1-\exp(-t/T_1)]$. The pip on the oscilloscope grows asymptotically toward its height appropriate to the new state.

Bloembergen first measured the spin-lattice relaxation time for distilled water at room temperature by taking motion pictures of the oscillograph screen as the pip grew. A semilogarithmic plot against time of the difference between the asymptotic height and the height at time t yields a straight line of slope $-1/T_1$. For the proton resonance at 29.0 Mc/sec, Bloembergen found $T_1 = 2.3$ sec, which Hahn[18] has measured to be 2.33 ± 0.07 sec.

A quite similar method may be employed for measuring T_1 in solids where the line is broad and it becomes necessary to use the "lock-in" amplifier. For example, Pake and Gutowsky[32] measured the relaxation time in a sample of ice at $-180°$C by saturating the spin system at a high r-f power level, then moving the external field several line widths off resonance. Periodically, they return to the resonant condition to measure, as quickly as possible, the amplitude of the resonance at a lower power level. This procedure is repeated from time to time until a sufficient number of points has been recorded to permit determination of T_1. Evidently T_1 must be the order of minutes to allow time to carry out the steps of this measurement. Further, a small error may be introduced by the amount of power absorbed during the measurement of each point.

16. Measurement of T_1 by Progressive Saturation

The extreme excursion of the lock-in output meter depends upon the signal generator voltage

in a way which is useful in determining relative values of T_1. To see this quantitatively, we need the result of Eq. (14.11) that the maximum lock-in deflection is, for the absorption curve,

$$D_{\max} = (\text{const.})G_r 4\pi Q |\mathcal{V}_0| a (d\chi''/dx)_{\max}, \quad (16.01)$$

where G_r is the receiver gain. To include in the discussion the effect of saturation, we require the value of χ'' from Eq. (11.09).

We first treat the situation in which T_1 is much longer than a modulation period and in which inhomogeneities do not fix line width. The denominator in Eq. (11.09) is then replaceable by its value at the center of modulation cycle. Upon placing the value of $(d\chi''/dx)_{\max}$ in Eq. (16.01) we have

$$D_{\max} = G_r |\mathcal{V}_0| (\text{const.})\pi Q a \\ \times [\tfrac{1}{2}\chi_0 \omega_0 T_2 (1 + \gamma^2 H_1^2 T_1 T_2)^{-\frac{3}{2}}]. \quad (16.02)$$

Suppose that, as $|\mathcal{V}_0|$ is increased, G_r is reduced so that the product $G_r |\mathcal{V}_0|$ does not change; this may be accomplished by adjusting the receiver gain to keep the detector current at the same value for each value of $|\mathcal{V}_0|$. Since H_1 is proportional to $|\mathcal{V}_0|$, $D_{\max}$ may be plotted against H_1 in order to find where the factor $(1 + \gamma^2 H_1^2 T_1 T_2)^{-\frac{3}{2}}$ reduces $D_{\max}$ to a given fraction of its asymptotic low power value. This measures T_1 provided T_2 is known from line width measurements and provided the proportionality factor between H_1 and $|\mathcal{V}_0|$ is known. The latter may be determined by direct measurement, by computation, or by calibration with a substance whose T_1 is known by direct measurement.

A graph of $D_{\max}$ against the saturation factor $s = \gamma^2 H_1^2 T_1 T_2$ is given in Fig. 9. Also shown is the BPP result for $D_{\max}$ against $\dot{s}$ in the event
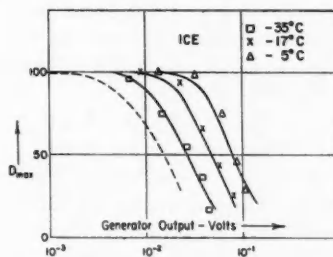

FIG. 10. Saturation curves measured for ice at various temperatures. The dashed curve illustrates the shape predicted by the Bloch theory.
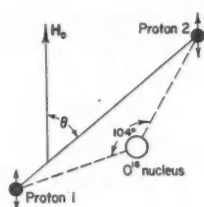
[32] G. E. Pake and H. S. Gutowsky, *Physical Rev.* **74**, 979 (1948).

Fig. 11. Magnetic environment of the hydrogen nuclei in a water molecule.

that T_1 is much shorter than a modulation period (β is taken to be the modulation angular frequency). Then the denominator in Eq. (11.26) changes during the modulation cycle (even though $a \ll 1$), and the algebra becomes more tedious. Those situations in which magnetic inhomogeneities fix line width will not be treated here. They are of importance for liquid samples (see page 691 of BPP).

In order to compare various substances, Bloembergen plots D_{max} against log $|\mho_0|$ obtaining curves such as those of Fig. 10. The dashed curve, which represents the factor $(1 + \gamma^2 H_1^2 T_1 T_2)^{-\frac{1}{2}}$, may be shifted to fit any experimental curve. It is significant that the theoretical curves of Fig. 9 do not differ greatly on a semilogarithmic plot and that, in experimental instances, the accuracy of the data may not warrant making the distinction between cases $T_1 \ll 1/\beta$ and $T_1 \gg 1/\beta$. The farther to the right the knee of a given saturation curve appears, the shorter is T_1 for a given T_2.

F. The Width and Structure of the Resonance

17. Sources of Line Broadening

Section 9 treated qualitatively several phenomena which broaden the resonance, all of which related to spin-spin interactions. It will shortly be seen that the spin-spin interaction reflects structural properties of the sample, and it is well to list other possible sources of broadening in order to know when a predominance of spin-spin effects is likely.

a. "Natural Width" for an Isolated Spin.—Equation (7.03), expressing the transition probability at any time for an isolated nuclear magnetic moment, does not describe an infinitely sharp resonance. It states that

$$C(\tfrac{1}{2} \to -\tfrac{1}{2}) \sim \frac{\theta^2}{[1-(\omega_0/\omega)]^2 + (\omega_0/\omega)\theta^2}.$$

Let $\omega = \omega_0 + \Delta\omega$ and make use of the fact that, even under the poorest experimental conditions, $\Delta\omega/\omega \ll 1$ for all ω at which appreciable absorption occurs. Then

$$C(\tfrac{1}{2} \to -\tfrac{1}{2}) \sim \frac{1}{1/\theta^2(\Delta\omega/\omega_0)^2 + 1}, \quad (17.01)$$

which has a half-width at half-maximum intensity of

$$\Delta\omega/\omega_0 = \theta = \tan^{-1}(H_1/H_0) \cong H_1/H_0.$$

Since the resonant field H_0 is usually several thousand gauss and H_1 is seldom as much as 0.1 gauss in absorption experiments, this "natural width" for the isolated spin is quite small.

b. Magnetic Field Homogeneity.—Even under the best geometric conditions, the magnetic inhomogeneity of magnet pole-face materials is normally sufficient to disperse the field through a range of 0.5 gauss or more in 10,000 gauss over a sample with about 1 cm² area projected on the field direction. The physicist must apparently content himself to live with inhomogeneities of at least several parts in 10^5 over the region of the sample, which sets an experimental minimum for the measured line width. Fortunately, this ever-present broadening will prove negligible in many applications.

c. Indeterminacy Broadening through the Relaxation Mechanism.—The lifetime of a nuclear spin state may be determined by relaxation processes under certain conditions, especially in liquid or gaseous samples, the thermal motions of which may cause spin-spin interactions to average out (see Sec. 20). The inverse spin-lattice relaxation time is (Sec. 8) the order of magnitude of the probability per unit time for an individual spin to be flipped by the "relaxing" local fields, and T_1 is the lifetime of the nuclear spin state when no other phenomenon limits the lifetime more severely. In this event, the Heisenberg relation specifies a spread in energy levels over a range

$$\delta U = \hbar/T_1, \quad (17.02)$$

which would spread the resonant frequency as determined from the Bohr condition over an amount

$$\delta\omega \sim \delta U/\hbar \sim 1/T_1. \quad (17.03)$$

Although it is quite common for other processes

to limit lifetimes more severely, BPP found several examples in which T_1 would determine line width in a perfectly homogeneous field. In glycerin, T_1 would fix line width when $T_1 > 10^{-3}$ sec, implying a frequency width $\delta\omega < 10^3$ sec^{-1} which corresponds to a width in gauss of $\delta H = \gamma\delta\omega \sim 1/30$ gauss. Evidently magnetic inhomogeneities will make it difficult to observe this width contribution, which is always present, however.

d. Saturation Broadening.—Equation (11.09) indicates that the half-width at half-maximum absorption is increased to

$$\delta\omega_{\frac{1}{2}} = (1/T_2)(1 + \gamma^2 H_1^2 T_1 T_2)^{\frac{1}{2}}, \quad (17.04)$$

when H_1 is sufficiently large to produce saturation effects. Care must therefore be taken, in investigations of spin-spin interactions, to see that the saturation factor $s = \gamma^2 H_1^2 T_1 T_2$ is substantially less than unity.

e. Electric Quadrupole Broadening.—Nuclei with $I > \frac{1}{2}$ may interact with the *gradient* of any electric field in which they find themselves. Such *electric quadrupole interactions* may broaden the line greatly[33] or even cause a pronounced splitting into component lines.[34] These effects will not be treated here in detail. However, it is known rigorously from theory that they cannot appear for nuclei of spin $\frac{1}{2}$, and it may also be shown that a lattice with cubic symmetry possesses an electric field such that no quadrupole splitting will occur. Most structural studies thus far have involved the proton or F^{19} resonances ($I = \frac{1}{2}$), and quadrupole broadening is not present.

18. The Spin-Spin Interaction between Pairs of Like Nuclei

It is common for the local magnetic fields produced at one nucleus of a solid by its neighbors to be the order of 10 gauss. Such spin-spin broadening usually swamps effects listed in the preceding section, of which magnetic inhomogeneity is normally the greatest. Values of these effects might be as follows in a representative experiment using the proton resonance:

Natural width	0.01 gauss
Inhomogeneity	0.5 gauss
Relaxation time width	0.03 gauss
Saturation broadening	Negligible
Quadrupole broadening	None.

We confidently expect spin-spin effects to dominate. However, this confidence is not always warranted, because substances which are solid by conventional standards may retain considerable molecular mobility so far as the relevant nuclear interactions are concerned.

For solids in which it is justifiable to think of each nucleus as being rigidly attached to some one lattice point, we write a classical expression for the magnetic field vector at distance $\mathbf{r}$ from a nucleus of magnetic moment $\mathbf{\mu}_1$

$$\mathbf{H}(1) = 3r^{-5}(\mathbf{\mu}_1 \cdot \mathbf{r})\mathbf{r} - \mathbf{\mu}_1 r^{-3}. \quad (18.01)$$

Should a second magnetic moment $\mathbf{\mu}_2$ be placed in the field of Eq. (18.01) at distance $\mathbf{r}_{12}$ from $\mathbf{\mu}_1$ the mutual energy of the pair is

$$U_{12} = \frac{\mathbf{\mu}_1 \cdot \mathbf{\mu}_2}{r_{12}^3} - 3\frac{\mathbf{\mu}_1 \cdot \mathbf{r}_{12}\mathbf{\mu}_2 \cdot \mathbf{r}_{12}}{r_{12}^5}. \quad (18.02)$$

To illustrate how this interaction can affect the nuclear resonance pattern, we give below a classical discussion of the broadening—splitting is a better word in this instance— of the proton line in a crystalline hydrate.

The hydrates are characterized by the rigid binding of water molecules into the crystal lattice. Since the common oxygen isotope has no spin or magnetic moment, each proton in a water molecule finds itself (Fig. 11) in a local magnetic field to which its nearest magnetic neighbor, the partner proton in its water molecule, is the chief contributor. Since the field [Eq. (18.01)] falls off as r^{-3} the effect of more

[33] R. V. Pound, *Physical Rev.* **73**, 1273 (1947).
[34] R. V. Pound, *Proc. Physical Soc.* (London) **61**, 576 (1948).
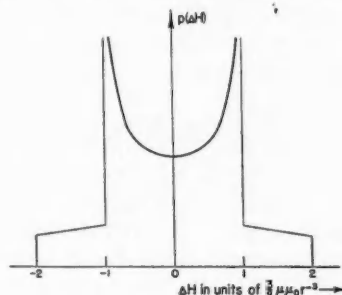
FIG. 12. Shape predicted for absorption line if the proton moments interact only within pairs.
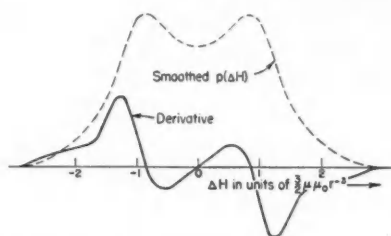
FIG. 13. Smoothed line shape obtained from the shape in Fig. 12. The derivative of the smoothed line is also shown.

distant protons is reduced. Each proton therefore resides principally in a magnetic field which is the vector sum of the external field H_0 and the field $H(1)$ of its partner. The field H_0 is usually of magnitude several thousand gauss, and the magnitude of the local dipole field is indicated by the value of $\mu\mu_0 r^{-3}$ which is 14.1 gauss for a proton at $1A(10^{-8}$ cm$)$. The magnitude of the sum of these two vectors is, to good approximation, the magnitude H_0 of the larger augmented by the component of the smaller along H_0. If H_0 is taken along z, the effective field at one proton is

$$H_{eff} = H_0 + H(1)_z. \qquad (18.03)$$

The direction of μ_1 is taken to be that of the static component of the precessing magnetic moment vector:

$$\mu_{1z} = \pm\mu\mu_0, \qquad (18.04)$$

the sign depending upon whether $m = \frac{1}{2}$ or $-\frac{1}{2}$ and upon the sign of g. Combining Eqs. (18.01) and (18.04) with Eq. (18.03) we obtain

$$H_{eff} = H_0 \pm \mu\mu_0 r^{-3}(3\cos^2\theta - 1) = \hbar\omega. \quad (18.05)$$

For each direction θ in the crystal, there is a pair of resonance lines separated in magnetic field by $2\mu\mu_0 r^{-3}|3\cos^2\theta - 1|$ according to this essentially classical picture.

We hasten to add that the quantum-mechanical computation[8] agrees insofar as the existence of a pair of lines for each θ is concerned, but finds the separation to be $3\mu\mu_0 r^{-3}|3\cos^2\theta - 1|$. The extra contribution of half-again as much splitting arises from the spin exchange process (Sec. 10) which involves the precessing components of μ; it is recalled that the foregoing classical argument took into account only the static components of the magnetic moment.

Experiments are reported in reference 8 which verify the existence of this splitting and the necessity for use of the $\frac{3}{2}$ factor. One such experiment examined the proton resonance in a single crystal of $CaSO_4 \cdot 2H_2O$, obtaining line shape as a function of magnetic field direction in the lattice. The crystal has a structure, determined by x-ray methods, which indicates that just two directions of the r vectors connecting protons are present in the crystal. Two pairs of lines were observed, at the predicted crystal orientations, and the maximum separation between each pair served further to check the crystal structure by measuring the distance between protons. The component fine-structure lines are not infinitely sharp, but have a half-width between two and three gauss contributed by the more distant protons in the neighboring water molecules; the maximum separation of a pair is 21.2 gauss.

For powder samples, which consist of a large number of randomly oriented microcrystals, one can compute the expected line shape by adding up the effect of these many crystals. The resultant shape $p(\Delta H)$ is plotted in Fig. 12. Of course, the function $p(\Delta H)$ does not include broadening effects contributed by more distant neighbors, and, if an empirical broadening is introduced to account for them, one obtains[8] the line shape of Fig. 13. In addition, this figure plots the derivative of the resonance, corresponding to the quantity determined directly by many experimental arrangements.

19. The Second Moment as a Measure of Line Width

Determination of the expected line splitting for an interacting nuclear pair is simple enough, but the addition of even one more nucleus to the interacting group increases the complexity greatly. The general problem involving several closely interacting nuclei is so tedious as to defy solution except for certain extremely symmetric configurations. However, just as the picture begins to appear blackest, we are saved by a theory of Van Vleck[35] which develops a quantitative relation between sample structure and line width. Van Vleck evaluates the so-called *second*

[35] J. H. Van Vleck, *Physical Rev.* 74, 1168 (1948).

moment of the distribution (line-shape function). This quantity is the mean value of the square of the frequency deviation from the center of the resonance, the average being taken over the shape function. The line shape, *versus* frequency, is taken to be a function $f(\nu)$ so normalized that $\int_{-\infty}^{\infty} f(\nu)d\nu = 1$. Then if ν_0 is the resonance frequency, the second moment is

$$\langle(\Delta\nu)^2\rangle_{Av} = \int_{-\infty}^{\infty} (\nu-\nu_0)^2 f(\nu)d\nu. \quad (19.01)$$

The nth moment is defined by replacing $(\nu-\nu_0)^2$ by $(\nu-\nu_0)^n$. The symmetry of the nuclear absorption curve about the resonant frequency, at least when ν_0 is large compared to the line width, requires that the odd moments vanish.

Van Vleck's calculation is essentially quantum-mechanical and will not be given here. Its ingenious feature is the expression of the second moment of the absorption line as the diagonal sum of a matrix. It is a theorem of matrix mechanics that this quantity does not depend on the wave functions used in formulating the matrix representation, and this permits Van Vleck to evaluate the diagonal sum using the simple properties of the wave functions which correspond to the individual noninteracting spins. The more complicated wave functions describing the interacting spins are therefore not required.

The result of Van Vleck's calculation is expressed below in gauss², appropriate to the technique of varying the magnetic field at fixed frequency.

$$\langle(\Delta H)^2\rangle_{Av} = (\tfrac{3}{2})I(I+1)N_s^{-1}g^2\mu_0^2$$
$$\times\sum_{j>k}(3\cos^2\theta_{jk}-1)^2 r_{jk}^{-6} + (\tfrac{1}{3})N_s^{-1}\mu_0^2$$
$$\times\sum_j\sum_f I_f(I_f+1)g_f^2(3\cos^2\theta_{jf}-1)^2 r_{jf}^{-6} \quad (19.02)$$
$$\langle(\Delta\nu)^2\rangle_{Av} = g^2\mu_0^2\hbar^{-2}\langle(\Delta H)^2\rangle_{Av}.$$
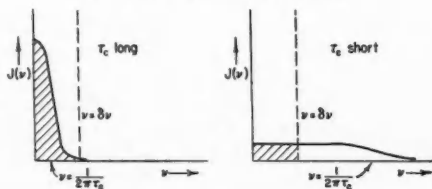
FIG. 14. Spectrum of the coordinates involved in internuclear interactions for two values of the correlation time.
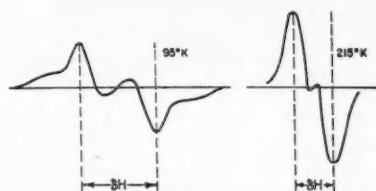
FIG. 15. Measured line shape derivatives for the proton resonance in 1,2-dichloroethane at 95°K and 215°K.

The following list defines the symbols in Eq. (19.02):

g,I Nuclear g-factor and spin for the nucleus having resonance

g_f,I_f Nuclear g-factors and spins for other nuclear species in the sample

r_{st} Length of the vector connecting nuclei s and t

N_s Total number of nuclei having resonance which are present in the sub-group (molecule, ion or atomic complex) to which the broadening interactions are considered to be confined in computing $\langle(\Delta H)^2\rangle_{Av}$.

Evidently the second moment for a powder sample can be obtained from Eq. (19.02) by averaging over all directions. Since the average value of $\cos^2\theta$ over a sphere is $\tfrac{1}{3}$,

$$\langle(\Delta H)^2\rangle_{Av}{}^{\text{powder}} = (6/5)I(I+1)N_s^{-1}g^2\mu_0^2$$
$$\times\sum_{j>k} r_{jk}^{-6} + (4/15)\mu_0^2 N_s^{-1}$$
$$\times\sum_i\sum_f I_f(I_f+1)g_f^2 r_{jf}^{-6}. \quad (19.03)$$

These formulas have been applied to measure the length of the N−H bond in the NH_4^+ ion of the crystalline ammonium halides,[36] and have been similarly applied to a number of organic molecules.[9, 37] The usefulness of nuclear resonance in such structural investigations is probably confined to those involving hydrogen, lithium, and other light elements which tend to resist x-ray methods by their failure to scatter with sufficient intensity.

[36] H. S. Gutowsky and G. E. Pake, *J. Chem. Physics* 16, 1164 (1948).
[37] E. R. Andrew, *J. Chem. Phys.* 18, 607 (1950).

FIG. 16. Atomic model of the 1,2-dichloroethane molecule.

20. The Effect of Nuclear Motion on the Line Width

The local magnetic field at a given nucleus involves a contribution similar to that in Eq. (19.06) from each neighbor, and, even in the quantum-mechanical treatment,[5] one needs to know

$$\sum_j |(1-3\cos^2\theta_{jk})r_{jk}^{-3}|^2, \qquad (20.01)$$

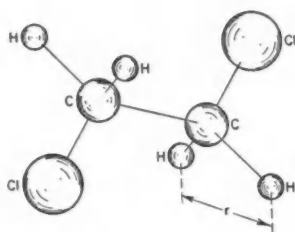as well as other functions of angle whose origin is more intimately associated with quantum-mechanical arguments. Whereas, for the rigid crystal lattice of Sec. 18 and 19, the expression (20.01) is merely a constant dependent on geometry, complications arise if θ_{jk} and r_{jk} are random functions of time. The statistical analysis, made in reference 5, requires that we consider the *spectrum* of the time average of Eq. (20.01), which is expanded as a Fourier integral

$$\langle \sum_j |(1-3\cos^2\theta_{jk}(t))r_{jk}^{-3}(t)|^2 \rangle_{Av} = \int_{-\infty}^{\infty} J(\nu)d\nu.$$

In reference 5 the intensity of this spectrum as a function of frequency is found to be

$$J(\nu) = K \frac{2\tau_c}{1+4\pi^2\nu^2\tau_c^2}, \qquad (20.02)$$

where τ_c, the *correlation time*, is a measure of the time in which the atomic configuration about the nucleus in question changes appreciably. The reciprocal $\nu_c = 1/(2\pi\tau_c)$ is thus an average rate at which molecular configurations change appreciably.

Note that $\int_{-\infty}^{\infty} J(\nu)d\nu = K$. Therefore, the area under the total spectrum is independent of τ_c. However, the line width is contributed only by *essentially static* interactions; that is, only the area of $J(\nu)$ near $\nu = 0$ is effective in determining the width. BPP takes the "static" portion of the

spectrum to be the area from $\nu = 0$ to a frequency $\nu = \delta\nu$ approximately equal to the line width for a rigidly frozen lattice.[38] In Fig. 14 the spectrum $J(\nu)$ is plotted for two values of τ_c. In each plot the total area under the spectrum is the same, although that portion of the area which is responsible for the line width is much smaller when τ_c is short. Indeed, the transition from the full width line, appropriate to a frozen lattice, to the narrow line characteristic of molecular motion is found to occur when $1/\tau_c$ has a value the order of the frozen-lattice line width (expressed as an angular frequency). This corresponds to a reorientation frequency the order of 50 kc/sec, which is extremely small compared to the frequencies ordinarily associated with atomic and molecular rotations or vibrations, about 10^{12} sec^{-1} or greater.

One can now predict what might be called the "expected normal behavior" of the line width in a solid. As the temperature rises, the width should presumably remain constant at a value of several gauss (or several tens of kc/sec) until, abruptly, the melting phenomenon heralds the breakdown of the forces which held the lattice rigid. The random molecular motions of the liquid will normally involve frequencies far greater than the 50 kc/sec threshold for line narrowing, and the line should be quite narrow. In fact, it is usually found that the resonance in a liquid is so narrow as to have an experimental width determined almost entirely by magnetic field inhomogeneities.

A number of substances have been found which fit well into the above scheme. However, many organic compounds exhibit, at temperatures well
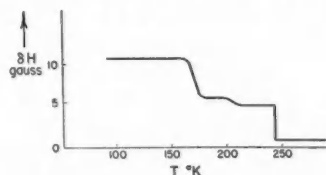
FIG. 17. Width between points of maximum and minimum slope of the proton resonance in 1,2-dichloroethane.

[38] This is not strictly correct, for BPP takes the line width itself to be the limit, which leads to an implicit determination. However, the simplification of always using the rigid lattice width does not impair our qualitative arguments, which indeed hold *a fortiori* if the "static" segment of the spectrum narrows as the width narrows. This correction also applies to the second graph of Fig. 14.

below the melting point, a narrowing from the frozen lattice width to an intermediate width greater than inhomogeneities might produce. We shall treat in some detail one such substance in which this behavior seems most easily understood.

21. Bond Distances and Internal Motion in 1,2-dichloroethane

At a temperature of 95°K the derivative of the proton line in CH_2Cl-CH_2Cl has the shape shown in Fig. 15; this shape is characteristic of the interacting pair (Sec. 18) and is presumably contributed by the proton pair on each end of the molecule (Fig. 16). At 210°K the line has a similar shape, but is only half as wide and is comparatively more intense (ordinates are not exactly to scale). The apparatus used in these studies[11] could hold only these two temperatures fixed, but it was possible to measure the separation δH between points of maximum slope as the sample warmed from 95°K. Figure 17 plots the result. One notes that the line narrows at 175°K to approximately half its former width, and finally it becomes quite sharp above the melting point near 240°K.

One can understand the broad line between 95°K and 160°K by noting that the splitting between the two peaks of the absorption curve corresponds very nearly to $3\mu\mu_0 r^{-3}$ (see Sec. 18) if $\mu\mu_0$ is the proton moment and r is the value expected for the proton-proton distance on the end of the 1,2-dichloroethane molecule. The estimate of r is based on the known $C-H$ bond distance of about 1.09A and the usual assumption of tetrahedral bond angles.[39] (See Fig. 16.) One then concludes that the molecules of the solid are at rest below 160°K, at least compared to rotational frequencies of 50 kc/sec.

The narrowing of the resonance may be explained by postulating that the molecule reorients itself about an axis perpendicular to the $H-H$ lines at the molecule ends (presumably the axis of least rotational inertia, which is slightly tipped toward the heavier Cl atoms from the $C-C$ bond). Such rotation, or discrete

jumping, must take place at frequencies exceeding 50 kc/sec, according to the arguments of Sec. 20, if the line is to be narrow at all. We now show a classical argument to indicate that this rotation will reduce the line width by a factor $\frac{1}{2}$.

Two resonance lines, separated by $3\mu\mu_0 r^{-3} \times |3\cos^2\theta-1|$ were found in Sec. 18 to arise from each orientation of the vector connecting protons. If the vector rotates about an axis making angle γ with the external magnetic field, one can use the addition theorem for spherical harmonics[40] to write

$$\langle 3\cos^2\theta-1\rangle_\psi = \tfrac{1}{2}(3\cos^2\gamma-1),$$

where the angles are defined in Fig. 18 and the brackets $\langle\ \rangle_\psi$ denote an average over the ψ motion. Then the two lines are split by

$$\tfrac{3}{2}\mu\mu_0|3\cos^2\gamma-1|$$

and, although γ differs from θ, it follows that a powder sample possesses all orientations of the rotational axis just as it contained all values of θ previously. Hence the powder line shape of Sec. 18 still appears, but with one-half its former width.

The word "rotation" has been used in this discussion in a general sense meaning reorientation about an axis. It is quite likely that the low frequency reorientations observed are relatively abrupt jumps of the molecule, or of one end of it, from one minimal position to another within a threefold or sixfold potential. These abrupt jumps may occur through a barrier penetration process analogous to the quantum-mechanical tunneling whereby, in radioactive decay of a nucleus, an alpha-particle suddenly appears on

FIG. 18. The angles used in an analysis of the rotating pair. OA =axis of rotation; OH =magnetic field direction; PP'=inter nuclear vector; θ=angle between internuclear vector and magnetic field; γ=angle between axis of rotation and magnetic field; ψ=azimuth angle of rotation of PP' about OA.

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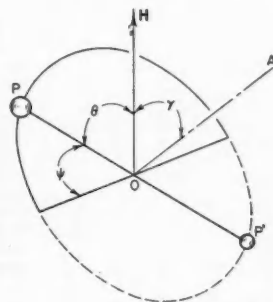
[39] The splitting observed does not quite correspond to the tetrahedral angle, and it may therefore present evidence for distortion of the bond angles as a consequence of the substitution of the larger Cl atom for a hydrogen atom.

[40] H. Margenau and G. M. Murphy, *The mathematics of physics and chemistry* (D. Van Nostrand, New York, 1943), p. 108, Eq. (3–61).

the other side of a potential barrier which, on a classical basis, it did not have the energy to surmount. According to this picture, such reorientation occurs about a definite axis at rates exceeding 50×10^3 sec^{-1} between 175°K and the melting point; it is replaced above the melting point by random relative motions of such rapidity that the $3 \cos^2 \theta - 1$ factor averages to essentially zero.

Comparison of experimental second moments with Van Vleck's theory, the detailed interpretation of line shape in certain instances, and the study of the narrowing of the resonance thus may lead to information concerning structural properties of solids. Use of nuclear resonance absorption for this purpose has been instituted by a number of investigators.[8-11,36] Since these methods require previous knowledge of nuclear spins and magnetic moments, the structural chemist and the solid state physicist join the nuclear physicist in his desire to know the magnetic and angular momentum properties of all the nuclei.

G. Nuclear Magnetic Relaxation

22. Lattice Motions and T_1

Even before the first radiofrequency experiment was performed, low temperature physicists displayed considerable interest in spin-lattice relaxation. It was hoped that adiabatic demagnetization, which had been successfully applied to the electronic spins of paramagnetic salts in order to reach low temperatures, could be applied

FIG. 19. Relaxation times as computed from the Waller theory applied to CaF$_2$. Experimental points obtained by Bloembergen are shown for comparison.

to nuclear spins. Since one step of an adiabatic demagnetization experiment involves waiting until the spins and lattice reach thermal equilibrium at a low temperature in a magnetic field, it becomes important to know how T_1 compares with the time available for performing the experiment.

Waller (1932)[17] assumed that the spin system in a magnetic field could give up quanta to the vibrations of the lattice. We know that the molecules of the crystal lattice possess vibrational kinetic energy of which the temperature is a measure. The spectrum of these vibrations may be expected to have, in general, components at the Larmor frequency of the nuclear spins in the magnetic field, and these components should induce transitions in accordance with a transition probability calculable from Eq. (7.01). However, when Waller's theory is applied to the F^{19} spins in the fluorite crystal lattice, the results of Fig. 19 are obtained.[20] Not only would these relaxation times render nuclear demagnetization experiments unfeasible, but recent measurements by radiofrequency absorption methods indicate values of T_1 many orders of magnitude smaller. These measurements for a clear fluorite crystal are plotted in the figure.

Agreement between theory and experiment, so far as spin-lattice relaxation times are concerned, was first obtained with liquid and gaseous samples (see BPP); the spectrum of random motions discussed in Sec. 20 is one of the major features of the theory. This spectrum describes the frequency distribution of components of the fluctuating local magnetic field produced at a given nucleus by its neighboring nuclear magnetic moments as they undergo random changes of their position relative to it. Evidently, this spectrum is closely related to the energy density function, $\rho(\nu)$, which appears in the expression (7.01) for the transition probability, being essentially proportional to it. The quantity $1/T_1$, which was found in Sec. 8 to be proportional to the transition probability, must therefore vary as $J(\nu_0)$.

$$1/T_1 = 2P \sim J(\nu_0). \qquad (22.01)$$

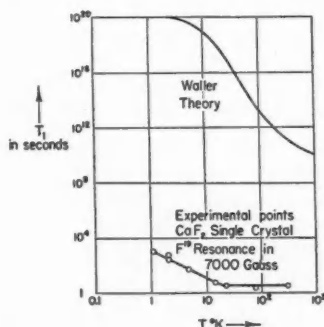Equation (20.02) can be combined with Eq. (22.01) to express the dependence of T_1 on τ_c,

the correlation time,[41]

$$T_1 \sim \tau_c^{-1}(1+4\pi^2\nu_0^2\tau_c^2). \qquad (22.02)$$

For $\tau_c \ll (2\pi\nu_0)^{-1}$, T_1 varies inversely as τ_c, whereas, for $\tau_c \gg (2\pi\nu_0)^{-1}$, T_1 varies directly as τ_c. Evidently T_1 has a minimum when $\tau_c \sim (2\pi\nu_0)^{-1}$.

Bloembergen performed experiments on glycerin, among other liquids, which support the theory. The correlation time was varied by changing the temperature, the results appearing in Fig. 20. Note that the position of the minimum varies with ν_0 in the predicted way. Bloembergen, Purcell and Pound also experimented with hydrogen gas, and found agreement with theory within the accuracy of their measurements.

It should be noted that the BPP theory reduces, in the event of long correlation time, to the Waller theory. In fact, there is a sizable class of molecular crystals, usually nonionic, which may be described by long correlation times. Thus the mystery surrounding relaxation in ionic crystals is still not dispelled by the BPP theory. Actually, no error of the theory lies in the method of calculation. If the lattice vibrations were the sole relaxation mechanism, the Waller theory would describe the experimental facts. The real problem centers around the discovery of the mechanism responsible for the spin-lattice interaction.

23. Other Relaxation Mechanisms

It was early recognized that in substances such as the paramagnetic hydrates the magnetic moments of electronic spins will interact strongly with the nuclear spins. These electronic spins, with magnetic moments the order of 2000 times greater than nuclear magnetic moments, interact more strongly with the lattice vibrations, the spectrum of which is in turn more intense at the higher Larmor frequencies appropriate to the electronic moments. For this reason the lifetime of an electronic spin state is short compared to

FIG. 20. Comparison of experiment with the BPP theory of relaxation in glycerin. It can be shown on theoretical grounds that the ratio of viscosity to absolute temperature should be proportional to τ_c^{-1}.

that for a nuclear spin state, and the randomly changing z-components of the electronic magnetic moments lead to fluctuating magnetic fields at nuclear spins throughout the lattice. In the first experiments on nuclear induction,[7] values of T_1 in liquids were reduced by dissolving paramagnetic ions in liquid samples, and the measurement of the He^3 magnetic moment in He^3 gas was made possible by introducing the paramagnetic molecules of oxygen into the sample to shorten the relaxation time.[25]

Investigators soon suspected that a minute, perhaps chemically unmeasurable, quantity of paramagnetic impurity in the lattice of an ionic crystal might well provide the relaxation mechanism. Bloembergen at Leiden[20] and Hatton and Rollin at Oxford[42] experimented with crystals containing various amounts of impurity; extrapolation back to amounts of impurity which might be present in "pure" crystals shows that such paramagnetic ions are quite likely the explanation for the discrepancy between theory and experiment in Fig. 19.

A similar effect of electronic magnetic moments has been observed by Rollin and Hatton,[43] who investigated spin-lattice relaxation in crystals containing color-centers. These color-centers are produced by x-radiation or by heating the crystal in a suitable metal-vapor atmosphere and are thought to be electrons trapped in the lattice. Evidently they should interact with the nuclear spins in such a way as to reduce T_1.

Heitler and Teller[44] in 1936 calculated semi-

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[41] Equation (22.02), although it demonstrates the essential features of the variation of T_1 with τ_c, is not strictly correct, for it omits a term involving the intensity of the spectrum at twice the Larmor frequency. The reasons for the inclusion of this term are clearly presented in BPP. We have taken the liberty of omitting it here because it simplifies the discussion without impairing in any way the qualitative picture we seek to develop.
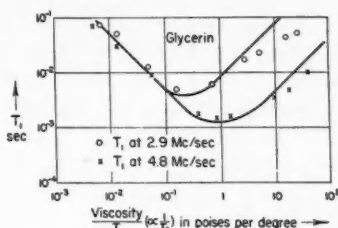
[42] J. Hatton and B. V. Rollin, Proc. Roy. Soc. A199, 222 (1949).
[43] B. V. Rollin and J. Hatton, Physical Rev. 74, 346 (1948).
[44] W. Heitler and E. Teller, Proc. Roy. Soc. A149, 152 (1936).

quantitatively the value of T_1 for the nuclear spins in a metal. The interaction for which they computed the transition probability is that between the nuclear spins and the conduction electrons in the high energy tail of the Fermi distribution. Experiments obtain an order of magnitude for T_1 which is in agreement with this calculation.

Inasmuch as the quest for low temperatures provided the initial impetus for interest in nuclear spin-lattice relaxation, a fitting note upon which to conclude this discussion is perhaps provided by the nuclear demagnetization experiment performed by Hatton and Rollin.[42] Having previously measured the value of T_1 to be 60 sec for the F^{19} resonance in a fluorite crystal at 1.2°K, they allowed the crystal spins to approach equilibrium in a field of 4000 gauss for 12 minutes at 1.2°K. The field was then dropped to 500 gauss, and the temperature of the sample was monitored by the strength of the F^{19} resonance in the 500 gauss field. (Since the signal strength depends upon the excess number, it measures the Boltzmann factor.) The temperature dropped to 0.17°K, and then rose to the original 1.2°K with the characteristic relaxation time previously measured.

* * *

This material was compiled to meet the needs of beginning graduate students working in nuclear resonance at Washington University. The establishment of this program of research and graduate training was made possible through joint ONR-AEC support and a grant from the Research Corporation.

The author wishes to thank Mr. E. J. Clarkin for drawing the figures.

List of Principal Symbols

$\boldsymbol{\mu}$ = magnetic moment vector

$\mathbf{p}$ = angular momentum vector

c = velocity of light *in vacuum*

M = mass of proton

e = charge on the proton

g = nuclear g-factor (dimensionless)

$\hbar = (2\pi)^{-1}$ times the Planck constant

$\mathbf{H}$ = magnetic field

$\mathbf{I}$ = nuclear spin vector (dimensionless)

m = magnetic quantum number (dimensionless)

I = nuclear spin (maximum value of m)

$\mathbf{L}$ = torque vector

ω_0 = angular frequency of Larmor precession

$U(m)$ = Zeeman energy of a magnetic moment in state m

$\mu_0 = e\hbar/(2Mc)$ = nuclear magneton

$\mu = gI$ = dimensionless number of nuclear magnetons: "the nuclear magnetic moment"

$\mathbf{B}$ = magnetic induction vector

$\mathbf{M}$ = volume density of magnetization

χ = magnetic susceptibility (dimensionless)

N = number of nuclear magnetic moments per unit volume

k = Boltzmann constant

T = absolute temperature, usually of the lattice

$N(m)$ = number of nuclei per unit volume in state m

χ_0 = nuclear Curie susceptibility

ω = angular frequency of incident radiation

$\mathbf{H}_1$ = rotating magnetic field

$\mathbf{H}_0$ = constant magnetic field vector

$P(m \rightarrow m')$ = transition probability per unit time

ν = frequency of radiation

$\rho(\nu)$ = energy density of radiation at frequency ν

$C(m \rightarrow m')$ = absolute transition probability

T_1 = spin-lattice relaxation time

T_S = temperature of the nuclear spin system

$W(p \rightarrow q)$ = total transition probability per unit time after detailed balancing

n = excess of nuclei per unit volume in lower Zeeman state (for $I = \frac{1}{2}$)

n_0 = equilibrium value of n for $T_S = T$

H_{loc} = magnetic field producing at lattice point by neighboring nuclear moments

A = energy absorbed per second per unit volume

χ', χ'' = real and imaginary parts of $\chi = \chi' - i\chi''$

M_0 = equilibrium value of z-component of nuclear magnetization

T_2 = time for a freely precessing component of nuclear magnetization to fall to e^{-1} of its value; inverse measure of line width

$\gamma = ge/(2Mc)$ = ratio of angular Larmor frequency to magnetic field

$x = T_2(\omega_0 - \omega)$ = dimensionless variable

$\mathcal{L}$ = complex inductance of coil containing nuclear sample

$\mathcal{Y}$ = complex admittance

$\mathcal{V}$ = complex potential difference

a = ratio of modulation amplitude (gauss) to line width (gauss)

G_r = gain of the receiver

D = deflection of "lock-in" meter

$s = \gamma^2 H_1^2 T_1 T_2$ = saturation factor

β = angular frequency of magnetic field modulation

ΔH = difference between H and its resonant value for applied radiofrequency ν

$p(\Delta H)$ = line shape for interacting pair

$f(\nu)$ = any normalized line shape on a frequency scale

$\langle (\Delta\nu)^2 \rangle_{Av}$ = second moment of line shape

τ_c = correlation time

δH = separation (in gauss) between points of extreme slope of the absorption curve.

Physicists In and Following World War II

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DURING World War II there was some talk in scientific circles of the inappropriate use of trained technical manpower by the military services. This attitude was accented, no doubt, by the reports of some in the military service who were incensed when detailed to work having no direct bearing on their professional experience. Much complaint was heard from civilians who at times were unable to obtain deferments for certain capable scientific workers in their employment.

Insofar as physicists were concerned, there now appears substantial evidence that surprisingly few professionally qualified physicists were in uniform at any time during the war. Furthermore, all but a very few of those in the armed forces were reasonably well assigned. This is amply borne out by the facts revealed in a special committee report of the American Institute of Physics to the Department of the Army, entitled: "Utilization of Physicists by the Armed Services During World War II."[1] This paper is a summary of the material in that report.

Never in the history of warfare has the role of science been so prominent as during World War II. Scientists who had known no previous association with the Armed Services nor had entertained any special interests in the implements of war were called upon to improve existing devices, discover new techniques, and develop new instrumentalities for the waging of war. Some scientists served as civilians in laboratories, in industry, or with the Armed Services in the field. Others, in uniform, were assigned to Army, Navy, or Air Force installations. Their achievements in developing better fighting tools for the men at the front are now well known.

In relatively new fields such as radar, proximity fuzes, guided missiles, and especially in the utilization of nuclear energy in the atomic bomb many successes were possible largely because of the unique qualifications of physicists. Members of this profession devoted their energies almost 100 percent during the war to development, production, and instructional matters directly connected with the war effort. There was little opportunity for the continuance of basic research. Consequently, there was an unusual drain on the stockpile of previously accumulated knowledge in the physical sciences.

Shortly after the end of the war some of the professional scientific societies independently had planned to survey the wartime activities of their members. While these plans were still in the initial stages the office formerly known as the Scientific Manpower Branch of the Research and Development Division of the War Department General Staff, now the Human Resources Section, Research Branch, Research and Development Group, Logistics Division, General Staff, U. S. Army, was formulating arrangements for a comprehensive study of the utilization of scientists during the war. Finally, a number of the scientific societies agreed to cooperate with the Department of the Army in a plan whereby each society would survey its members with regard to their wartime experience. The objectives of these surveys included an analysis of the educational and professional backgrounds of the scientists, their current occupations and the utilization of their technical specialities by the Armed Services. The American Institute of Physics agreed to assume the responsibility for conducting the survey of physicists.

The basis for the results of this survey was a questionnaire sent out by the Institute in November, 1946, to about 8000 persons then included on the rolls of its member societies or on its subscription lists. It is certain that some of these persons (perhaps 10 percent) were not professional physicists. As will be noted later, the presence of these nonphysicists served to render

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[1] Privately circulated.

TABLE I. Work status of physics respondents (1946–47).

Work classification	Number	Percentage of total
Industry	2542	40.7
Academic	2405	38.5
Govt. agency	750	12.0
Military service	38	0.6
Student	431	6.9
Retired or unemployed	81	1.3
Total	6247	100.0

less valid some of the generalizations arrived at in this study.

All of the questions except two were prepared so that they could readily be tabulated and scored by machine methods. These two questions (numbers 14 and 15) deal with the nature of the respondent's military service and their comments on the effectiveness with which they were utilized while in uniform. Because of the nature of these questions and to secure uniformity of analysis, it was thought best to apply a rating code to the answers in terms of the personal judgment of two or three persons. Many of these ratings were independently checked by each person of the committee which prepared the report.

Results of Survey

The following material has been arranged to give, first of all, a picture of the work status and the educational attainments of respondents at the time (1946–1947) the questionnaires were executed. Then will be shown the distribution of physicists in various wartime activities and finally a review of the utilization of the professional qualifications of those in the armed services in the period 1942–1945.

A. Over-all Figures on Those Reporting.—Out of approximately 8000 questionnaires sent out to individuals associated with the American Institute of Physics, about 75 percent were returned and were used in the compilation of data (Fig. 1). Of those so responding, 96 percent were males. Although 10 percent of the total reported military service during the war, only 6 percent considered such service as their *principal* war work.

About 90 percent of the respondents were physicists. A few were associated with other

branches of science or perhaps maintained only a lay interest in physics; but at least they were in some respects identified with the field under survey, being subscribers to physics journals.

B. Work Status of Respondents.—Table I shows the distribution of physicists in six categories of work status as of 1946–47. Nearly 80 percent of those reporting were evenly distributed between academic work and industry; 12 percent were in government agencies; most of the remaining 8 percent were students.

C. Educational Background.—It is a matter of interest to know something of the educational background of the respondents. This information, which had never been ascertained before as regards the physicists of this country, is given in Fig. 2. The titles refer to the highest degrees received *in physics.* Apparently 75 percent have college degrees in physics; of these, nearly half (48 percent) have attained the doctorate. It is not a little surprising to find the numbers having the master's and the bachelor's degrees essentially the same. Many of the 735 persons reporting no academic degrees *in physics* are nonphysicist subscribers to magazines on the rolls of the AIP. Many have doubtless received degrees in other major fields of science.

D. Formal Education in Physics for Each Category of Work Status.—Figure 3 presents, for each of the six categories of work status, the distribution of individuals according to the highest academic degree attained in physics.
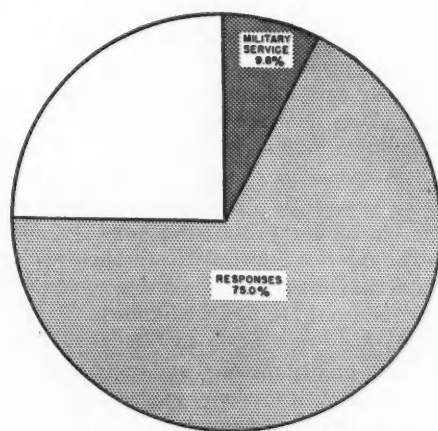

FIG. 1. Questionnaires sent and responses.

Comparing the various categories, those in schools, colleges, and universities not only have the greatest number (1239) of Ph.D.'s, but also have the greatest proportion (51 percent) of Ph.D.'s in their own ranks. The government agencies also have highly trained talent, with 41 percent Ph.D.'s. Industry has 27 percent Ph.D.'s. In each of these three groups, i.e., academic institutions, government agencies, and industry, about one of every five persons has attained the master's level.

E. *Principal War Work of Physicists.*—The portion of the questionnaire from which the data underlying Table II were taken called for the "Principal type of work in which engaged between February 1, 1942, and September 1, 1945." Here the respondent was invited to identify his *principal* war activity with one of seven fields, as follows: (1) government sponsored research; (2) industrial other than government sponsored; (3) teaching; (4) government research; (5) government, other than research; (6) armed services; (7) student. All of these activities, even including that of "student," may be considered to have a direct bearing on the war effort. If one were to single out government research, government sponsored research, and industrial work as having the most direct bearing on the nonoperational aspects of the war program, it will be noted that these three fields embrace two-thirds of the total respondents. Of the remaining third, about 64 percent were

TABLE II. Principal work during war of physicists and AIP subscribers.

Principal war work	Number of respondents	Percentage of total
Govt. sponsored research	1789	38.3
Industrial, other than govt. sponsored	1540	24.3
Teaching	1368	21.6
Govt. research	889	14.0
Govt. other than research	169	2.7
Armed services	391	6.2
Student	184	2.9
Total	6330	100.0

teaching. No doubt much of this teaching was of pupils in the various Army and Navy training programs—an activity certainly allied very closely to the war effort.

Of the 2406 respondents in educational work at the time of the survey, about 50 percent were teaching during the war and 27 percent were engaged primarily in government sponsored research. Of the remaining 23 percent, about half were associated with the government or were engaged in government research. Over 80 percent of the 2542 in industry reported wartime activity in industry or in government sponsored research. It is interesting that among 430 who were students at the period surveyed, 37 percent were, during the war, active in government research.

It was observed in comparing the current work status of respondents and their war activities that in categories other than "student" and "retired" about 50 percent of those in any group were in the same type of work during the war as they followed after hostilities ceased. Whichever may be the cause and which the effect, it may be said that the transition resulted in surprisingly little dislocation.

F. *Utilization of Physicists in Military Services.*—About 10 percent of the total respondents reported having been in the Armed Forces during the war, and these were about evenly divided between the Army and the Navy (Fig. 4). A very small number were with the Marines, Coast Guard, Waves, etc. As for method of entry (Fig. 5), approximately 40 percent received direct commissions, 25 percent were drafted, 20 percent were in the Reserves, and 15 percent enlisted voluntarily.
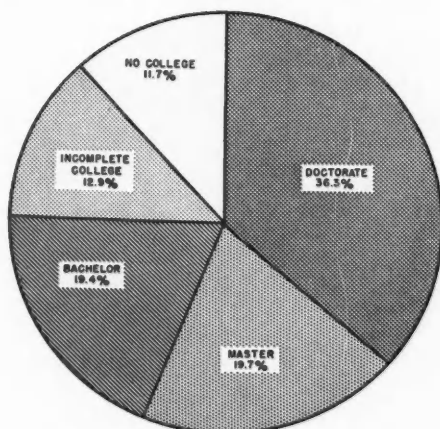

FIG. 2. Highest degrees in physics received by AIP members and subscribers.
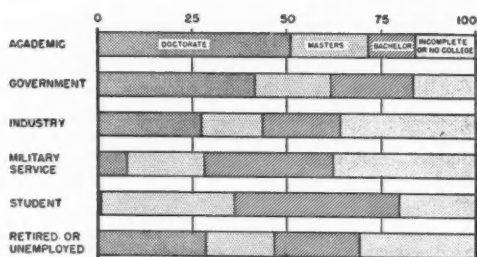
Fig. 3. Education of physicists and AIP subscribers in various types of work status.

Of primary importance are the data which suggest the degree to which the professional training and experience of those reporting Military Service were utilized in the Armed Forces. This information was taken from items 14 and 15 in the questionnaire. From a study of these items each respondent was grouped in one of the five classifications of utilization given in Table III.

Although these percentages are shown graphically in Fig. 6, it is well to see the actual counts given above since one must bear in mind that no great numbers of persons are here involved. It is noted that over 50 percent are classed in No. 1. utilization, and less than 6 percent in No. 5. If one considers utilizations No. 1 and 2 as indicating satisfactory use of specialized talents, it is gratifying to find 70 percent in these two groups.

As is to be expected, those receiving direct commissions in the Armed Forces were best utilized, whereas those drafted fared most poorly (Fig. 7). One should bear in mind, however, that of 606 reporting military service, only 34 appear to fall in No. 5 utilization. It is of incidental interest to note in Fig. 8 the distribution of doctors, masters, and bachelors in respect to method of entry into the armed services. There were reported 7 Ph.D.'s drafted, 8 enlisted, 38 entered through the Reserves, and 53 given direct commissions.

In Fig. 9 one finds the utilizations grouped according to degree of formal education in physics. It is noteworthy that No. 1 utilization embraces the largest number of individuals in each group. The actual figures showed that the Ph.D.'s embraced about one-fifth of the individuals in utilization No. 1 and also in No. 2; but less than one-tenth in Nos. 4 and 5. The

masters constituted about 20 percent and the bachelors about 40 percent in groups No. 1, 2, 4, and 5. This suggests that the Ph.D.'s were more satisfactorily utilized than those attaining lower degrees. One should be cautious, however, in assigning too much significance to these figures since so few individuals are involved in many of these data.

Summary of Results Concerning Physicists

The highlights of information gleaned from some 6000 questionnaires from physicists and AIP subscribers have been given in the foregoing paragraphs. In summarizing the picture so drawn, it is intended to bring together these facts with the circumstances which, at the time, may have influenced the results. Certainly the whole process of placing professional personnel in wartime activities did not develop merely by chance, but took form through active participation by private as well as public agencies. The wartime procurement and utilization of manpower trained in physics were a primary concern of the American Institute of Physics and the Office of Scientific Personnel.

In the postwar period, 1946–1947, the survey indicates that the great majority of physicists, about 80 percent of them, were more or less equally divided between industry and academic employment. Of these, the schools and colleges got the larger share of Ph.D.'s. This fact is
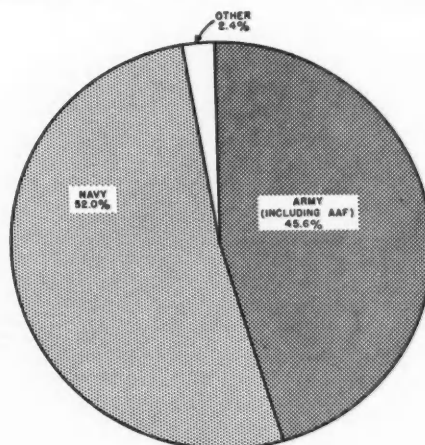

Fig. 4. Distribution of 623 physicists in the Armed Services in World War II.

especially significant in the realization that of all physicists having attained a college degree, about half had received the doctorate. Figures are not immediately available to compare these observations with prewar conditions; and therefore the trend in comparative employment of physicists by industry and by schools is not specifically known. In college placement offices, however, the continued demand by industry for men with the higher degrees in physics is abundantly evident at the present time. The U. S. Navy and other government offices, in view of their needs for civilian physicists, are giving every encouragement to scientific employees to upgrade their special qualifications by academic training while on the job.

During the war there were few activities in which a physicist could participate professionally without a more or less direct bearing on the war effort. According to the survey about 10 percent of those reporting had military service, but only 6 percent considered such service as their *principal* war work. Roughly two thirds of the physicists were directly connected with war research and development. The remainder were mostly college and university teachers and students. It is gratifying that the reconversions, so to speak, from wartime to peacetime jobs show that the major portion of those *now* teaching were teaching during the war. Likewise, most of those now in industrial research were so employed during the war. This of course does

TABLE III. Utilization of professional training and experience of physicists while in military service.

Group	Number	Percentage	Degree of utilization
1	322	53.3	Well utilized as physicist at proper level of competence
2	96	15.8	Utilized as physicist for at least half of time in service
3	15	2.4	Utilized in field collateral with physics during most of service
4	139	23.0	Utilized as physicist or related work during short portion of service
5	34	5.5	Not used as physicist or in related field at any time during service
Total	606	100.0	

not imply that the nature of their teaching or their research was the same both during and after the hostilities.

As for the utilization of professional qualifications of those in the military service, the survey suggests that 70 percent were reasonably well placed. In this respect the Ph.D.'s fared somewhat better than the others. It is noteworthy that out of some 608 reporting military service, only 34 felt that their special abilities were not utilized at all. This picture, at least so far as physicists are concerned, appears much brighter than one would have surmised from some of the cloakroom talk during the war.

Statistically those physicists who were in the armed services were fairly well utilized. It must be recognized that the services have need for some physicists in uniform and that some physicists, in time of war, are more interested in that type of service. The record of utilization is better, no doubt, than it would have been had not the American Institute of Physics, the National Research Council, and other agencies assisted the War Department in locating and reassigning misplaced physicists. Such assistance was applied in the cases of a quarter or more of all of the physicists in uniform.

The statistics of utilization in the armed services do not paint a black picture when looked at alone. When the value of the average contribution to the war effort per man is considered, however, the record within the service cannot be said to compare favorably with the over-all civilian record. This conclusion is borne out by the statistics of the report and even more impressively by the records of civilian research on
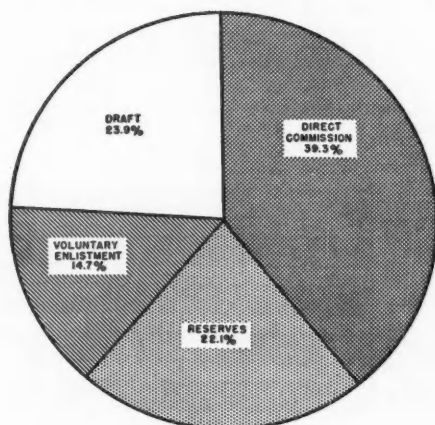

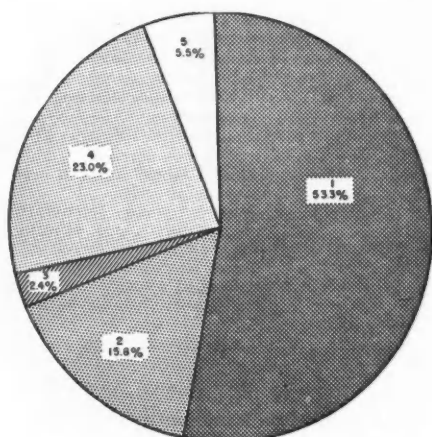FIG. 5. Method of entry of physicists into Armed Services in World War II.

FIG. 6. Utilization of physicists in uniform
in World War II.

nuclear energy, radar, proximity fuzes, subsur-
face warfare, and other important areas.

It is a good thing the number of physicists in
uniform was limited, not only for the reason just
referred to, but also because of the fact that the
services were forced into drastic measures to
make the most of the limited numbers of physi-
cists they had. Without the force of this shortage
the utilization would not have been as good as
it was.

These observations point up the belief that
the efforts of the Selective Service System to
transfer physicists from civilian to uniformed
status were largely misdirected. No suggestion is
offered of improper motivation in the operations
of the System. It is intended rather to make
clear how the Selective Service, in some aspects,
worked counter to the best interests of the war
effort.

In the first place the draft as it operated
placed an implied stigma on a large number of
the younger physicists who were performing an
essential service to their country. Second, the
burden of proof was placed on the physicist to
justify his civilian war service and this was done
repeatedly and with increasing insistence. From
the standpoint of the war effort it would have
been more logical if a case had had to be made
to take him away from his civilian war service.
Third, a great amount of time of directors of
even the most important war research projects

was necessarily spent in holding their staffs to-
gether and keeping the rate of turnover within
bounds. Negotiations with draft boards, with
state and national headquarters, and with other
government agencies having delegated powers,
were time consuming, difficult because of the
variety of personalities involved, often carried on
at a distance, and frequently on a level of under-
standing quite inadequate to the purpose.

The power of decision in draft cases almost
never, throughout the war, resided together with
technical comprehension, access to pertinent
classified information, and an over-all awareness
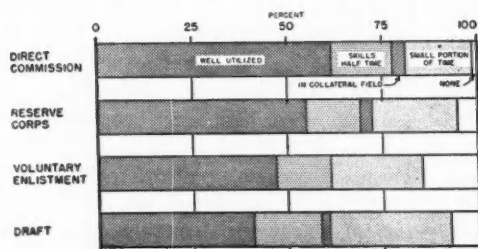of war needs. In a technical war, especially in

FIG. 7. Utilization of physicists in uniform in terms of
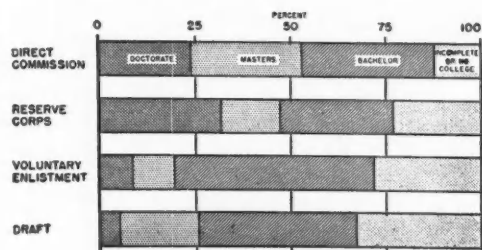method of entry into armed services.

FIG. 8. Distribution of physicists in uniform having various
degrees in terms of method of entry into services.
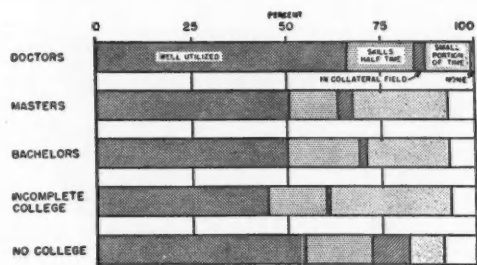
FIG. 9. Utilization of physicists in uniform in groups
having various academic degrees.
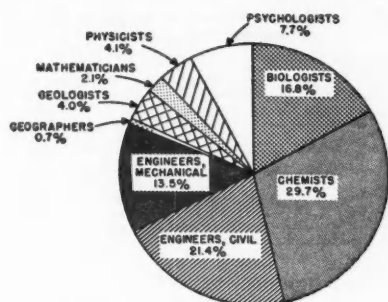
FIG. 10. Distribution of scientists and engineers
in uniform in World War II.

the case of the physical sciences, it is difficult
to see how there could be any adequate substi-
tute for such a conjunction of responsibility,
competence, and information.

In view of these conditions one might well
observe that with so many hands groping around
in the manpower barrel it is remarkable not
that things turned out so well, but that indeed
they worked out at all!

Comparisons with Other Sciences

In addition to the data presented above with
respect to physicists the Department of the
Army secured the assistance of the professional
societies in eight other fields of science and en-
gineering in conducting similar surveys of the
military experiences of a large number of their
members. These societies distributed nearly
135,000 questionnaires; 69,000 were returned;
15,000 of the respondents were in uniform dur-
ing World War II. A report on these studies has
been published by the Department of the Army.[2]

In Fig. 10 is shown the distribution by disci-
plines of the 15,000 scientists and engineers in
the armed services during the war who furnished
usable data for this survey. Only about 10,000
of this group reported military service as their
principal war work. In 1945 only 329 of these
people remained in the armed services.

These scientists and engineers entered the
military service by the methods shown in Fig. 11.
This may be compared with the corresponding
data for physicists shown in Fig. 5. The physi-

[2] "Scientists in Uniform, World War II," U. S. Govern-
ment Printing Office, 1948.

cists enjoyed a larger proportion of direct com-
missions while a smaller percentage of physi-
cists entered the services by voluntary enlist-
ment. The percentages of physicists drafted and
entering from the reserves are about the same
as those in the larger group.

Using the same objective scale as that em-
ployed for the physicists to report the degree of
effective utilization of their technical specialites
14,782 scientists and engineers gave the dis-
tribution shown in Fig. 12. These individuals
were distributed in nine major scientific fields
as shown in Table IV. Physics and psychology
reported the best utilizations while the biologists,

FIG. 11. Method of entry of 15,000 scientists and
engineers into armed services.

FIG. 12. Utilization of scientists and engineers in
uniform in World War II.

TABLE IV. Utilization by the Armed Forces
of scientists from eight fields.

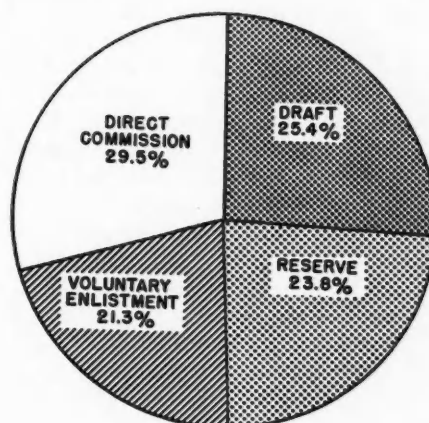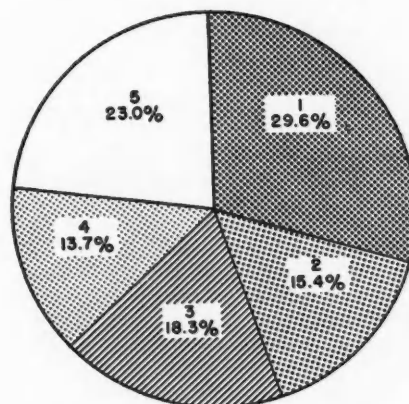| Degree of utilization	Biology	Chemistry	Civil engineering	Mechanical engineering	Geology	Mathematics	Psychology	Physics
				Scientific Fields				
1	22.5%	23.9%	33.7%	34.4%	5.0%	35.5%	45.6%	52.2%
2	11.5	12.6	17.4	14.1	21.6	11.6	26.5	16.9
3	11.4	20.2	21.8	25.9	23.3	10.9	14.0	2.4
4	11.3	16.0	14.4	11.8	13.9	18.2	8.7	22.5
5	43.3	27.3	12.7	13.8	36.2	23.8	5.2	6.0
Number	2830	3967	3202	2025	605	303	1168	581
Percent of grand total	19.1	26.9	21.7	13.7	4.1	2.0	7.9	3.9

TABLE V. Comparison of utilization of engineers
in uniform and as civilians.

Utilization category	Percentage of engineers in uniform	Percentage of engineers as civilians
1	33.9	74.9
2	16.2	11.2
3	23.4	8.2
4	13.4	2.7
5	13.1	3.0

geologists, chemists, and mathematicians were most poorly utilized in their professional fields.

The civil and mechanical engineers who participated in this study were asked to compare their professional utilization while in uniform with that while they were civilians. Of the 5200 engineers in uniform and the 9600 who worked as civilians and who reported on this item the data shown in Table V were obtained. The marked contrast in the effectiveness of utilization is all the more noteworthy in view of the fact that the civilian groups include many of those also in the military group, since most of the engineers in uniform also had civilian experience during the emergency before or following their tenure in the armed services.

In the item requesting recommendations concerning utilization of scientists by the armed services in the event of any future national emergency some 4000 replies were received. These were broken down into two categories and are shown in Figs. 13 and 14.

These suggestions for better utilization of scientists in uniform as well as the general results of the survey are being studied by the Human Resources groups in the Army, Navy, and Air Force and in the Research and Development Board. The American Institute of Physics and the other professional societies participating in these investigations are using these data to plan and make recommendations concerning the better mobilization of scientists in the event of another national emergency. It is hoped that this may serve to minimize the lack of liaison between science and the Armed Services which existed prior to the recent world wars. The failure of the military, in the prewar period, to appreciate fully the applicability of scientific and technical developments to the instrumentalities of war prevented their full and immediate exploitation. This circumstance is also largely responsible for much of the dissatisfaction with the assignment of scientists in the Armed Services.

It seems reasonable to urge that scientists and particularly the scientific and engineering societies take the leadership in making plans now during peace(?)time for a realistic mobilization of scientists and science in any future war. If organized science is not willing to assume such responsibilities, individual scientists should not

FIG. 13. Suggestions for better utilization of scientific manpower in a national emergency.
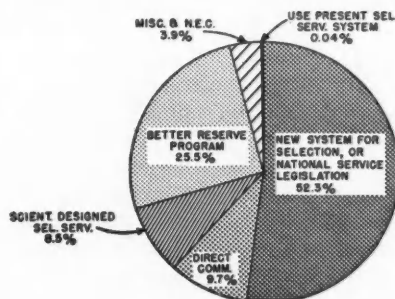
FIG. 14. Suggestions for better organizational structure for utilization of scientists and engineers in war time.

be too critical of the plans made by other agencies created to do this work during the stress of a national crisis. It is encouraging to observe the leadership of physicists and their organizations in current efforts to have this science accept the large responsibility to which its rapid development has led in national and world affairs.

Grateful acknowledgment is made for the assistance rendered in this study by the late Cleveland Norcross, formerly Assistant Director of the American Institute of Physics. He and Dr. Henry A. Barton made valuable contributions in the preparation of the report of the Institute to the Department of the Army from which the data contained in this paper were taken.

Landmarks in the Theory of Magnetism*

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IN the style of recent Sunday newspapers, I might well entitle my talk "The theory of magnetism in the first half of the twentieth century," for I am not going to discuss how the Greeks, Romans, and Chinese theorized about magnets and lodestones, or even go back to the immortal work of Oersted and Ampère in the nineteenth century in showing that electric currents create magnetic fields. The pioneer attempts of Weber and Ewing at a molecular theory were also in the nineteenth century, but the modern theory of magnetism may be considered as starting with Lorentz's electron theory and his interpretation of the Zeeman effect, which came at practically the turn of the twentieth century. I shall try to show that the history of the theory of magnetism in our century is pretty much the history of extranuclear physics in this era, and that each important advance in the knowledge of atomic structure has had far-reaching repercussions for the subject of magnetism. By dragging in nuclear magnetic resonance, molecular beams, etc., I might even progress quite far in using magnetism as a telescope for viewing progress in nuclear physics, but that would be stretching things pretty far for an hour's talk, and so I will exclude all nuclear effects in what I say today.

Langevin's Theory (1905).—Although by 1900 the electron theory of matter was becoming established, the big inaugural date in the modern theory of magnetism is 1905, for in that year Langevin published his classic paper on paramagnetic and diamagnetic susceptibilities—a matter of only some 15 pages, and a prime object lesson that one need not publish voluminously in a field to make an outstanding contribution to it. Also it illustrates the fact that a great theory is usually a basically simple one. Langevin's formula for the paramagnetic susceptibility is one of the most rudimentary examples of Boltzmann statistics.

If we have a magnetic dipole of moment μ making an angle θ with an applied field H, then its energy in this field is $-\mu H \cos\theta$, and its Boltzmann probability of location in an element of solid angle $d\omega$ is equal to

$$\frac{e^{\mu H \cos\theta/kT} d\omega}{\int\int e^{\mu H \cos\theta/kT} d\omega}.$$

Here the denominator is inserted in order to make the total probability integrate to unity. If N denotes the number of molecules/cc, the magnetic moment per unit volume is

$$M = \frac{N \int\int \mu \cos\theta e^{\mu H \cos\theta/kT} d\omega}{\int\int e^{\mu H \cos\theta/kT} d\omega}.$$

Evaluation of the integral gives

$$M = L(\mu H/kT), \tag{1}$$

where $L(x)$ is the so-called Langevin function

$$L(x) = \coth x - (1/x). \tag{2}$$

Usually the energy of the dipole in the field is small compared with the equipartition measure kT. Under these circumstances the exponentials may be developed as power series in $\mu H/kT$, and a simple calculation shows that the leading term in the susceptibility χ, i.e., in the ratio of moment to field strength, is

$$\chi = M/H = N\mu^2/3kT. \tag{3}$$

This result accorded nicely with the discovery that Pierre Curie had made before Langevin's paper that the susceptibility of many paramagnetic substances is inversely proportional to the absolute temperature, a principle commonly termed Curie's law.

Curie had also found that the susceptibility of diamagnetic materials, i.e., of substances with negative χ, is independent of temperature. Langevin showed that this fact could be explained neatly by invoking Larmor's theorem that in a magnetic field, the electrons in an atom all precess about the field with a common angular velocity $He/2mc$. The sense of the precession is

FIG. 1. Saturation intensity of magnetization as a function of temperature. The abscissa is the ratio of the temperature to the Curie temperature and the ordinate is the ratio of the saturation intensity of magnetization to the value of this intensity at the absolute zero. The classical theory of Weiss predicts that in these units the curve should have the same form, labelled $S = \infty$, for all materials. The other curves are the quantum-mechanical modifications appropriate to a spin quantum number S.

such as to create a moment opposite to the field, and hence a diamagnetic effect. The resulting susceptibility can be shown to be

$$\chi = -(Ne^2/6mc^2)\sum_i \overline{r_i^2}, \tag{4}$$

where bar denotes the time average, and the summation is over the different electron orbits of radii r_i. If the molecule has a nonvanishing permanent moment μ, then the paramagnetic or positive term of Eq. (3) caused by the Boltzmann alignment effect far overshadows the diamagnetic one of Eq. (4), and the diamagnetic contribution will enter only as a small correction to the susceptibility. A truly diamagnetic material, however, has $\mu = 0$, and only the diamagnetic or negative part of the susceptibility is present. From the observed diamagnetism, information can be deduced concerning $\sum_i r_i^2$, and hence concerning the size of atomic radii. In the early days of the kinetic theory of matter, this was one of the best ways of deducing that atomic diameters are of the order 10^{-8} cm.

On the whole, remarkably little has been added to the theory of diamagnetism since Langevin's paper. Quantum mechanics tells us how to evaluate the mean square radii as quadratures involving the Schrödinger wave function. In molecules, in distinction from atoms, there are corrections to formula (4) arising from the effect of high frequency matrix elements. There are some tricky quantum effects associated with the diamagnetism of free electrons. Also, some anomalously large radii are encountered in the susceptibility of graphite, showing that the electrons must circulate here around more than one atom, and there is always the tantalizing possibility that there is some connection between such anomalously large diamagnetic susceptibilities and superconductivity. From some, but not all standpoints, a superconductor may be regarded as merely a body whose susceptibility is $-1/4\pi$, so that the magnetic induction $B = H + 4\pi M$ vanishes. By and large, however, the theory of diamagnetism is a rather simple affair compared with that of para- and ferromagnetism. It has to a large extent been annexed by chemists, and as this is a talk primarily for physicists, I shall not allude to it further.

Weiss' Theory of Ferromagnetism (1907).—Langevin's original paper gave models for dia-
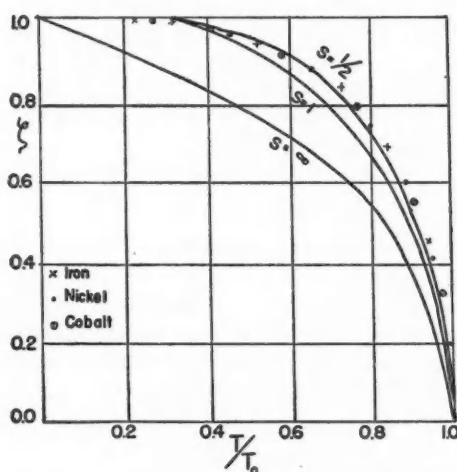
and paramagnetism, but did not include one for ferromagnetism. You will recall that a ferromagnetic body is one with an enormous positive susceptibility, whereas a paramagnetic body is one with just a moderate positive susceptibility, though a hundred or so times larger than the diamagnetic effect. Only two years after Langevin's article, another French physicist, Pierre Weiss, showed that a beautiful phenomenological explanation of many of the salient facts of ferromagnetism could be obtained if the field entering in the argument of the Langevin function (2) be taken to be not just the applied field H, but rather the latter augmented by a "molecular field" which is proportional to the intensity of magnetization, so that

$$M = L\left(\frac{\mu(H+qM)}{kT}\right). \tag{5}$$

Weiss showed that a formula of this type will explain many phenomena in a semiquantitative way. It may be, in fact, compared with van der Waals' equation in thermodynamics as a remarkably shrewd analytical description—semitheoretical, semiempirical—of the physical processes occurring. It likewise has a law of corresponding states, one of its greatest triumphs. If one takes as ordinate the ratio of the saturation magnetization to that at $T=0$, and as abscissa the ratio of the temperature to the Curie temperature, then in these reduced units a universal curve is predicted for the variation of magnetization with temperature. The agreement of this curve, labelled $S=\infty$, in Fig. 1, with experiment is gratifyingly good for so simple a theory, though not as close as for the subsequent quantum-mechanical refinements of the Weiss theory indicated by $S=\frac{1}{2}$ and $S=1$.

Difficulties (*1910*).—At this stage one is inclined to be optimistic, but when one examines the theory more critically, certain very critical difficulties are revealed, and so the decade or so beginning about 1910 may be termed the dark age in the history of magnetism. One of the difficulties is that it was impossible to account for the magnitude of the molecular field. Presumably this field is to be regarded as a manifestation of a cooperative effect wherein the elementary magnets strive to set themselves parallel—an effect

which Ewing had envisaged in less quantitative form in the nineteenth century. However, it is quite easy to calculate the classical electromagnetic interaction of an ensemble of similarly oriented dipoles, for this problem is the basis of the classical demagnetization factor. In this fashion one can show that the factor q cannot exceed $8\pi/3$, and what is worse, it should be shape-dependent, vanishing for a sphere, and actually becoming negative for a fat, chunky specimen. Weiss, however, required the factor q to be of the order 10^5, and practically independent of shape.

Another difficulty was that experiments by Barnett and others, on magnetization by rotation, begun in 1909, consistently yielded a gyromagnetic ratio, i.e., a ratio of magnetic moment to angular momentum, almost twice as large as that $e/2mc$ predicted by Lorentz' electron theory.

Absence of Magnetism with Pure Classical Statistics (*Bohr, 1911*).—An even more serious difficulty was that if Boltzmann statistics were applied systematically to all degrees of freedom, no magnetism at all would be obtained. Langevin obtained a nonvanishing result only because of a subtle inconsistency. He used the Lorentz electron theory for the diamagnetic term. On the other hand he treated the permanent dipole moment μ as a purely phenomenological entity and little if any attempt was made to correlate it with the angular momentum of the molecule. In 1911 a young Danish physicist by the name of Niels Bohr showed, I believe as his doctor's dissertation, that in any dynamical system to which Boltzmann statistical mechanics can really be applied, the diamagnetic and paramagnetic terms cancel—a most disconcerting result. Essentially the same point was proved, by a somewhat different method, by a Dutch physicist, Miss van Leeuwen, in 1919. A simple example will illustrate the germ of the idea underlying their proofs of zero total susceptibility. Suppose that the molecular model consists of a moving charge e constrained to remain at a fixed distance r from an attracting center. According to the electron theory of Lorentz the ratio of magnetic moment to angular momentum is $e/2mc$, so that the magnetic dipole moment is

$$\mu = (e/2mc)mr^2\dot{\theta}.$$

The kinetic energy of the model in question is $T=\frac{1}{2}mr^2\dot{\theta}^2$. According to classical Boltzmann statistics, this energy should have the mean value kT, and hence the mean square dipole moment for an ensemble is

$$\langle\mu^2\rangle_{Av}=\frac{e^2r^4}{4c^2}\langle\dot{\theta}^2\rangle_{Av}=\frac{1}{2}\frac{e^2r^2}{mc^2}kT. \qquad (6)$$

If we use Eq. (6) in Eq. (3) and if we remove the bars and summation from Eq. (4), which is legitimate since only a single charge of a fixed radial distance is involved, we do indeed obtain equal and opposite values for the expressions (3) and (4), and the total susceptibility is zero.

Of course, it will immediately be objected that Boltzmann statistics cannot be applied at all to a real Rutherford atom, for the Boltzmann probability is overwhelmingly large that the electron be infinitely close to the nucleus because there the potential energy is negatively infinite. Instead of assuming this nonconverging continuous distribution of orbital radii, Langevin supposed that somehow the molecule had a fixed value of the magnetic moment, and hence presumably a fixed value of the angular momentum. Hence, Langevin was quantizing angular momentum, without realizing it, just as in Molière's *Bourgeois Gentilhomme*, Monsieur Jourdain has been writing prose all his life, without appreciating it, and is overjoyed to find that he has been doing anything so highbrow.

Although the way out of the difficulty of Bohr and van Leeuwen is to be found in the quantum concept, it should be mentioned that on the whole the elegance and simplicity of their proofs of the absence of magnetism in classical statistics has not been properly appreciated. The literature abounds with formulas for the diamagnetism of free electrons in classical theory. It is, however, meaningful to apply classical statistics to free electrons, and the only correct result can be zero. The reason that nonvanishing results are obtained in certain classical papers is that the boundary electrons are improperly handled. When quantum effects are included, there is indeed a small diamagnetic effect from free electrons, but this vanishes for $h=0$.

Period of the Old Quantum Theory (1913–1926). —The advent of the Bohr quantum theory of the atom in 1913 did not take the mystery out of the molecular field, but did show that the earlier objections of Bohr himself to Langevin's result were not relevant. The essence of the Bohr theory of atomic structure is that the angular momentum of the atom has only certain quantized values, and so any dilemma based on the application of the continuous distributions of Boltzmann statistics ceases to be a case in point.

The period 1913–1926 contained many attempts to account quantitatively for the observed susceptibilities. Most of these papers, based on absurd types of quantization or on molecular rotations of a variety not actually encountered in solids, now seem as ridiculous as the hobble skirt and other *mores* of the era. The period may be described as one of spectroscopic instability, for different expressions for the susceptibility are obtained according as spatial quantization does, or does not take place, and depending on the direction selected for the axis of quantization. In consequence, baffling contradictions were obtained.

Quantum Mechanics and Spectroscopic Stability (1925–1926). —The years 1925 and 1926 marked the arrival of quantum mechanics, and this placed the theory of magnetic susceptibilities on a rational basis. The mean value of $\cos^2\theta$, with true quantum mechanics, comes out $\frac{1}{3}$ as in classical theory regardless of the choice of the axis of quantization—a result sometimes termed the principle of spectroscopic stability.

Electron Spin (1925–1926). —Practically concomitant with the advent of quantum mechanics was the development by Uhlenbeck and Goudsmit of the concept of electron spin. It is hard to tell whether quantum mechanics or electron spin has done more for the theory of magnetism. In the Dirac electron the two are, of course, inextricably intertwined. Even without spin, quantum mechanics would furnish a logical, consistent foundation on which to build a theory, but the magnetic moment of the electron plays a peculiarly important part in the subject of magnetism. The spin has only a half quantum of angular momentum, but a full Bohr magneton $he/4\pi mc$ of magnetic moment. Hence its gyromagnetic ratio has twice the normal value. We immediately see that with electron spin, the difficulty in the Barnett experiments disappears—provided, of

course, that most of the magnetic moment comes from the spin.

Wide, Narrow, and Intermediate Multiplets.— The simultaneous advent of quantum mechanics and electron spin suddenly threw wide open the door to the calculation of magnetic susceptibilities. Before discussing applications to specific atoms or molecules, it is well to outline the general conditions under which Langevin's formula is valid in quantum mechanics. There are certain multiplet structures which measure the strength with which the spin and orbital angular momenta are coupled either to each other or to the crystalline environment. Ordinary spin-orbit multiplets and crystalline Stark splittings are examples. In order that Langevin's formula for the paramagnetic susceptibility may be valid, it is necessary that these multiplets have intervals which are either very wide or very narrow compared with kT. On the other hand, it is immaterial how the rotational and vibrational energy levels of the molecule are spaced relative to kT, for rigid rotations and small elastic vibrations do not appreciably affect the size of the resultant magnetic moment.

Monatomic Gases and Vapors.—First, let us consider the confirmation of the theory in gases. All monatomic gases are diamagnetic, and so there is no chance to test the theory here. There are some nice measurements on alkali vapors, but these difficult experiments serve more as a check on the values of the heat of dissociation than on the magnetic theory, for the latter is much better established than the expressions for the amount of polymerization.

Theory for O_2 and NO (1927).—Now let us turn to diatomic molecules. There are only two common diatomic gases which are paramagnetic, namely, oxygen and nitric oxide. The former has multiplets which are very narrow compared with kT. The latter has a multiplet structure comparable with kT. It is lucky that Pierre Curie measured the temperature variation in oxygen rather than nitric oxide, for otherwise he might never have been led to enunciate Curie's law. The spin multiplets in oxygen have a wavelength of about $\frac{1}{2}$ cm, and during the war, and afterwards, have been of interest because of the resulting attenuating effect of the oxygen in the atmosphere on the propagation of exceedingly short radar waves. However, the energy interval involved in such a multiplet is only about 1/100 as large as kT at room temperatures, and so the multiplet structure should have virtually no effect on the magnetic susceptibility. Hence oxygen should obey Curie's law, and indeed probably does so within the experimental error. The quantitative magnitude of the susceptibility of O_2 can be calculated very simply since the oxygen molecule is in a $^3\Sigma$ state. This means that there is no mean orbital moment, and so the susceptibility should obey the so-called "spin-only" formula. This is by now such a standard expression that it is worth while taking a little time to discuss it. The "spin-only" formula is obtained by taking the square of the magnetic dipole moment to be $\mu^2 = 4\beta^2 S(S+1)$, so that the Langevin expression (3) becomes

$$\chi = 4N\beta^2 S(S+1)/3kT. \tag{7}$$

Here β is the Bohr magneton $he/4\pi mc$, S is the spin quantum number, and so $S(S+1)$ is the square of the spin angular momentum, measured in multiples of $h/2\pi$. The factor 4 occurs in Eq. (7) from squaring the anomalous factor 2 in the gyromagnetic ratio for spin.

The O_2 molecule has $S=1$, and with this value of S, the theoretical value of the molar susceptibility (i.e., the susceptibility per unit volume multiplied by the volume of a gram mole) at 293°K is 3.43×10^{-3}. The agreement with experiment is good. Some of the measured values are

3.35×10^{-3} (Curie, recalibrated)
3.45×10^{-3} (Bauer and Piccard)
3.31×10^{-3} (Onnes and Oosterhuis)
3.33×10^{-3} (Soné)
3.48×10^{-3} (Wills and Hector)
3.34×10^{-3} (Lehrer)
3.42×10^{-3} (Woltjer, Coppoolse and Wiersma).

Now let us turn to nitric oxide. The NO molecule is in a $^2\Pi$ state, and furnishes the standard illustration of deviations from Curie's law caused by a multiplet structure which is comparable with kT. The width of the NO doublet is 122 cm^{-1}, and so is about $0.6kT$ at room temperatures. In studying the dependence on temperature, it is convenient to take as the ordinate not the susceptibility itself, but rather the so-
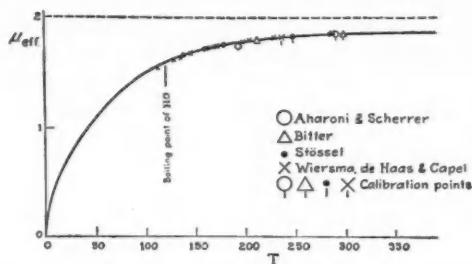
[FIG. 2. Effective Bohr magneton number of nitric oxide as a function of temperature.

called effective Bohr magneton number, defined by

$$\mu_{EFF} = [3kT\chi/N\beta^2]^{\frac{1}{2}}, \tag{8}$$

where χ is the experimental value of the susceptibility. If the susceptibility obeys Curie's law, the effective Bohr magneton number will be constant. The departures of the curve in Fig. 2 from a horizontal straight line therefore measure the deviations from Curie's law. When I first calculated the temperature variation, I had a very healthy respect for experimental difficulties, and supposed that it would be impossible to obtain sufficient vapor pressure to verify the theoretical curve below room temperatures. However, a variety of experimenters, working to progressively lower temperatures as shown in Fig. 2, have confirmed the predicted dependence on temperature, as well as the absolute magnitude of the moment.

Degeneracy and Magnetic Moment. Kramers' Theorem (1930).—Before turning to polyatomic molecules, I will state without proof two fundamental theorems of quantum mechanics connecting the existence of a magnetic moment with the degree of degeneracy. A level is said to be degenerate when it is composed of two or more sublevels which coincide, i.e., stick together perfectly. An easy calculation shows that in order for a level to have a mean magnetic moment in quantum mechanics, it is necessary for it to be degenerate, i.e., to be a multiple level. It can be shown that if one uses the Schrödinger orbital equation without spin, any sufficiently asymmetrical electric field will lift the degeneracy, i.e., pry all levels apart. On the other hand, a very remarkable theorem of Kramers shows that if spin is included, a twofold degeneracy will

always persist, regardless of how asymmetrical the electric field, provided the molecule contains an odd number of electrons.

Polyatomic Molecules and Free Radicals.—Now let us try applying these theorems to polyatomic molecules, i.e., molecules composed of more than two atoms. Usually the atoms are rather unsymmetrically arranged, or if there is symmetry, it turns out that the lowest orbital level is nondegenerate. Hence in polyatomic molecules, the orbital moment is quenched, and the susceptibility should conform to the spin-only hypothesis. Ordinarily, if the molecule has an even number of electrons, the spins will pair off, and the molecule will be in a singlet state, and diamagnetic. Oxygen is somewhat exceptional among even molecules in this regard. For a molecule with an odd number of electrons, complete pairing is impossible, and there will be paramagnetism.

The predicted conformity of the susceptibility of odd polyatomic molecules to the spin-only formula (7) with $S=\frac{1}{2}$ has been confirmed by experimental measurements on NO_2 and a few other gases, but the number of odd molecules which occur normally in the gaseous state is very severely limited. A much more fertile field in which to test the theory is in solutions containing such molecules. Here the theory of magnetism has furnished a valuable indicator for the chemist studying complicated molecules. Often he is interested in whether a group of atoms occurs as a so-called free or unsaturated radical with an odd number of electrons, or is polymerized into even groups with completely paired spins. In the former case there should be para-
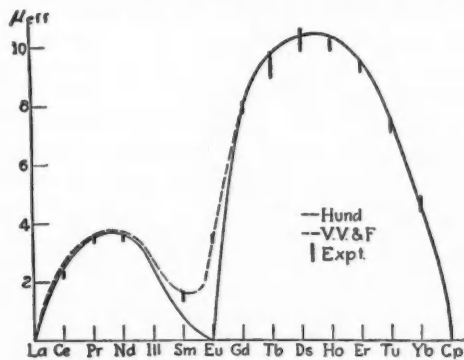

FIG. 3. Effective Bohr magneton numbers for the rare earths.

magnetism—in the latter diamagnetism. By measuring the absolute value of the susceptibility it is even possible to study the amount of polymerization. For instance, Asmussen has shown that the yellow salt obtained from the alkaline oxidation of hydroxylamine disulphonic acid is only feebly paramagnetic in the solid state, but in blue aqueous solution this substance conforms to the spin-only formula within 10 percent or so, showing that free radicals occur in the latter. There is thus shown to be a transformation of the form

$$(KSO_3)_4N_2O_2 \text{ (solid)} \leftrightarrow 2(KSO_3)_2NO \text{ (solution).}$$
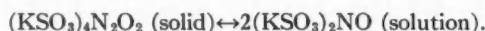
Another interesting example is the curious color centers found in certain photoelectrically excited solids. These have a susceptibility conforming substantially to the spin-only formula with $S=\frac{1}{2}$. The ingredient responsible for the coloring is thus an ion or free radical with an odd number of electrons.

Rare Earths (1925–1929).—Usually one would not think of applying to solids a physical theory developed primarily for gases. However, in the case of solid rare earth salts at room temperatures, the magnetic theory for isolated atoms can actually be employed with good results. The reason that one can do this is that the magnetism of rare earth ions, usually trivalent, arises from
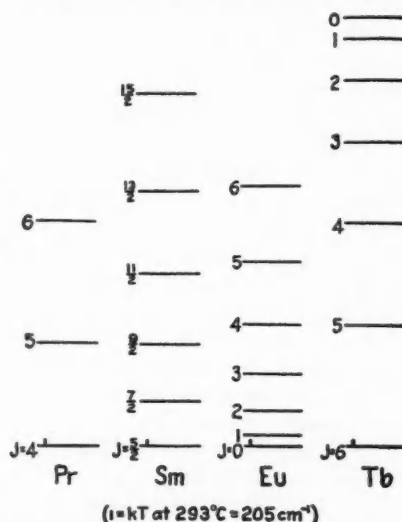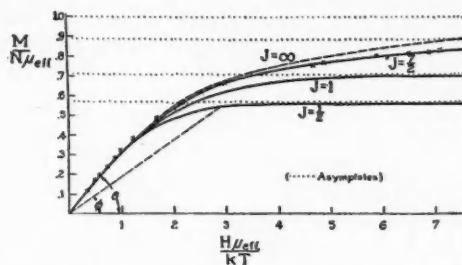
FIG. 5. Saturation curves for various values of the inner quantum number J. The scale of ordinates is so adjusted that all curves have the same initial slope, corresponding to the same susceptibility in weak fields.

incomplete $4f$ shells, and their electrons are so deeply sequestered in the interior of the atom that they are well screened from interference by their neighbors. Before one can apply the theory, it is, of course, necessary to know the quantum numbers or spectroscopic classification of the ground state. This can be determined from Hund's rule that the deepest state is that of maximum multiplicity admitted by the exclusion principle for the state in question, and of the maximum resultant azimuthal quantum number L compatible with this multiplicity. In 1925, a few months before the advent of quantum mechanics, Hund showed that except for two elements he could successfully explain the observed susceptibilities of the series of rare earth ions. This accomplishment was, indeed, one of the greatest triumphs of spectroscopic classifications. As you know, before quantum mechanics, Landé had developed his semiempirical theory of the g-factor, and what Hund did was to take the square of the magnetic moment in the Langevin formula (3) to be $\mu^2 = J(J+1)g^2\beta^2$, where J is the inner quantum number. However, I have not placed the theory for the rare earths earlier in the historical sequence because it was not until 1929 that Miss Frank and I showed that samarium and europium, the two ions which initially appeared recalcitrant to theory, could be brought into line. The original theory of Hund had assumed that the multiplet intervals are wide compared with kT, but for these particular ions this is not true. When the more complicated theory for intermediate size multiplets is developed, it is found that all fourteen rare earths obey the theory in a remarkable fashion, as

FIG. 4. Comparison of multiplet interval with kT for certain rare earths.

shown in Fig. 3. Here the vertical lines give the range of the experimental values of the effective Bohr magneton number. The unrevised Hund theory gave zero for europium, and an excessively low moment for samarium.

It is to be emphasized that incomplete inner shells are a prerequisite for magnetism in the solid state. Incomplete outer shells will not do, for their spare electrons will be snatched by other atoms to make the solid an ionic compound whose ingredients are composed entirely of closed shells. The Bohr-Pauli theory of atomic structure tells us where in the periodic table the incomplete inner shells are to be expected, and Fig. 3 is a graphical illustration of a nicely worded sentence by Niels Bohr: "A consideration of the magnetic properties of the elements within the long periods gives us a vivid impression of how a wound in the otherwise symmetrical inner structure is first developed and then healed as we pass from one element to another."

Figure 4 illustrates why it is not allowable to treat the multiplet interval as wide compared with kT in samarium and europium. Here the size of kT is illustrated by the vertical line. In europium, the lowest level is separated from the next highest one by an interval of the same order as kT. On the other hand, in terbium, which resembles europium in being in an ^7F state, the inversion of the multiplet for a shell over half full makes wide intervals come at the bottom of the energy level diagram, and only one level is relevant at ordinary temperatures.

I have emphasized that Curie's law should be obeyed unless there are multiplets comparable with kT. Consequently, samarium and europium should be the only rare earths exhibiting marked deviations from Curie's law at room temperatures. This prediction is in accord with experiment.

The theory for the rare earths is also confirmed in very difficult measurements on the gyromagnetic ratio made by Sucksmith, and by experiments on paramagnetic saturation. So far I have discussed only the dependence on temperature and not that on field strength. The theoretical dependence on field strength can be tested in a paramagnetic, in distinction from ferromagnetic, material only by using very low temperatures, for in a paramagnetic substance there is no molecular field to "boost" the magnitude of the argument of the Langevin function. Therefore, with any experimentally realizable field, this argument will be very small compared with unity, and the moment, hence, an uninteresting linear function of field strength, unless the temperature is reduced to the helium region. Because of the fact that in quantum mechanics one sums over a discrete number of orientations rather than integrates over a continuum as in classical statistics, the saturation curve is given by a different formula from that of Langevin. The so-called Brillouin function must be used, whose form depends on the value of the inner quantum number J. Figure 5 shows that from the saturation measurements it would have been possible to deduce that the spin of the Gd^{+++} ion is 7/2 had this not been known from Hund's theory. It should be mentioned that the Leiden physicists who made the saturation experiments in 1915 and 1923, before Hund's theory of spectral terms was available, were unwittingly wise in using $Gd_2(SO_4)_3 \cdot 8H_2O$, for gadolinium is the only rare earth ion which is in an S state, and which thus has such small crystalline Stark effects that it can be treated like a gas even at helium temperatures.

Pauli's Theory of the Feeble Paramagnetism of Conduction Electrons (1927).—Usually one thinks of statistical mechanics as a tool responsible for developments in paramagnetism. However, in the theory of conduction electrons in solids, the reverse is true, for the study of magnetism has guided the use of statistical mechanics in this field. Pauli's paper in 1927 explaining why conductors are only feebly paramagnetic marked the beginning of the quantitative application of Fermi-Dirac statistics in problems involving conduction electrons, and actually anteceded and stimulated the well-known sequence of papers by Sommerfeld and his students, which appeared about a year later. The rare earth salts which I have been discussing are insulators whose magnetic electrons are cloistered deep inside the atom. Since the vagabond electrons of a conductor have a free spin and are thus in a certain sense odd molecules, one would expect according to Langevin's formula that conducting metals such as the alkalis would be highly paramagnetic, with a susceptibility inversely pro-

portional to temperature. Actually the susceptibility is only about a thousandth of what one would compute with this formula, and is more or less independent of temperature. Pauli showed, however, that this anomaly is explained away when one treats the electrons according to the Fermi-Dirac rather than Boltzmann statistics. The Fermi-Dirac formulas can be regarded as a manifestation of the Pauli exclusion principle, which Charles Mendenhall used to describe so aptly as "a piece of social legislation to avoid the overcrowding of electrons." According to Boltzmann statistics, practically all electrons should have their spins parallel to the field at low temperatures because of the resulting lower energy. However, such alignment involves their doubling up, two to a state, a gregariousness forbidden by Pauli, unless the orbital quantum numbers are made different and the translational energy hence unduly high. The situation may be likened to that of a hotel on a warm day in which all occupants insist on privacy, and would prefer to be on the north side of the house. However, they would have to climb additional flights of stairs if they left the south side vacant and so they fill both sides approximately equally. There is thus only a small excess of parallel over antiparallel spins, and hence only a feeble paramagnetism, which is furthermore reduced by one third because of a small amount of diamagnetism for free electrons which exists in quantum mechanics in distinction from classical theory.

Heisenberg's Theory of Ferromagnetism (1928).—The other extreme from feeble paramagnetism is, of course, ferromagnetism. In 1928 Heisenberg removed the mystery from the Weiss molecular field. He showed that because of the constraints exposed by the Pauli exclusion principle, the quantum-mechanical exchange forces between atoms are formally equivalent to a large coupling between spins. It is thus now possible to understand how the parameter q of the Weiss field can be as large as 10^5. It is, in fact, easier to understand the magnitude of the field than its sign, for to date theory has had at best only qualitative success in predicting the conditions under which the exchange integral is positive and ferromagnetism hence possible.

After the initial wave of papers that followed Heisenberg's theory, there have been few, if any articles that can be classed as landmarks in the theory of the ferromagnetism of a single domain. However, our understanding of the domain structure of ferromagnetic materials is steadily improving. Unless the applied field is very powerful, a ferromagnetic body is to be conceived of as an assembly of small domains whose directions of magnetization do not coincide. These domains are separated from each other by what are termed Bloch walls, in which the direction of magnetization gradually changes. The theoretical basis for the existence of these walls was first developed by Bloch in 1932, and put into more quantitative form by Landau and Lifschitz in 1934. In the last few years the existence of the elementary domains has been unequivocally demonstrated in the beautiful powder patterns obtained first by Bitter and much more completely by Williams and Bozorth.

Antiferromagnetism (1932).—An important offshoot of Heisenberg's theory of ferromagnetism is the subject of antiferromagnetism. The first papers in this subject appeared in the early thirties, but it is only in the last decade that the field has attracted much attention, for of late the instances found experimentally have increased materially. An antiferromagnetic body is one in which the exchange integral, and hence the constant q of the Weiss field, is negative, and in which the exchange forces are so large that at sufficiently low temperatures an ordered arrangement is favored in which the spins are alternately parallel and antiparallel to the field. Above a certain critical or Curie temperature, this staggered ordering, which is incidentally accompanied by an anaemic sort of hysteresis, disappears rather suddenly, just as does the parallel ordering for a true ferromagnetic. Consequently the variation of the susceptibility with temperature has the rather peculiar form shown in Fig. 6.

The great complexity of the theoretical problems posed by either ferromagnetism or antiferromagnetism is to be stressed. Even if the atoms are effectively in S states, so that one has the "spin-only" model used originally by Heisenberg, the equations can be solved only approximately. The formulas yielded by the so-called first approximation in a series development in the ratio of the exchange integral J to kT work quite well, and are remarkably like those

of the Weiss theory, the main difference being that the Brillouin function occurs in place of the Langevin one, as shown in Fig. 1. The second or so-called Gaussian approximation leads to worse results than does the first order one. Recent revisions of the work of Opechowski indicate that the third and fourth approximations are not as unsatisfactory as had generally been supposed, but in any case the convergence of a series development in J/kT is rather slow. Fortunately, other methods of approximation are available, the so-called spin-wave theory of Bloch at low temperatures, and the Bethe-Peierls-Weiss method at high temperatures. Even if the secular problem for the spin-only problem could be solved accurately, this would not be enough, for spin-orbit interaction must be invoked to explain anisotropy, remanence, hysteresis, and all the properties of the domains and Bloch walls. At this point I shall therefore leave ferromagnetism in the hands of the engineers and experimental physicists, who are continually producing better ferromagnetic materials and powder patterns, and revert for the rest of my talk to the theoretically easier subject of paramagnetism.

Influence of the Crystalline Field on Paramagnetic Susceptibilities (*1932*).—Although the theory for gases applies quite well to rare earth salts of high magnetic dilution at room temperatures, marked deviations are found at low temperatures. For some years prior to 1932 it had been generally suspected that the Stark effect
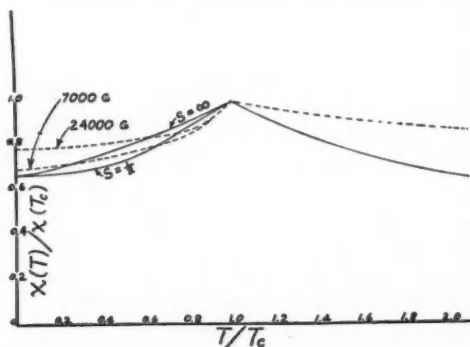
FIG. 6. The susceptibility of an antiferromagnetic material as a function of the "reduced temperature," i.e., the ratio of T to the Curie temperature T_C. The solid curves are theoretical, while the dashed lines give the experimental results of Bizette, Squire, and Tsai on MnO.

arising from the electric fields exerted by the neighboring ions was responsible for the discrepancies, but it was not until then, I believe, that a quantitative theory based on this idea was first developed by two British physicists, Penney and Schlapp, who at that time were research fellows at the University of Wisconsin. Their calculations were greatly facilitated by the use of Bethe's group theory of crystalline Stark patterns. The clue to successful results was found in the assumption of a crystalline field of dominantly cubic symmetry. We had tried to fit the observed susceptibilities by using a crystalline potential of the form

$$V = Ax^2 + By^2 + Cz^2 \qquad (9)$$

and were getting nowhere. Then one day Penney found that he could make the theory work by employing a cubic crystalline potential of the form

$$V = D(x^4 + y^4 + z^4). \qquad (10)$$

The magnetic data thus show that the potential (10) predominates over (9) even though (10) represents a higher order member than does (9) in the series development of the potential in the ratio of orbital radius to interatomic distance. Figure 7 shows that the variations with temperature for $Pr_2(SO_4)_3 \cdot 8H_2O$ and $Nd_2(SO_4)_3 \cdot 8H_2O$, two typical rare earth salts, are rather well represented by the theory. The ordinate in Fig. 7 is the reciprocal of the susceptibility per gram ion. The solid curves are calculated theoretically, and the points are experimental. Notice the different behavior for $Pr_2(SO_4)_3 \cdot 8H_2O$ and $Nd_2(SO_4)_3 \cdot 8H_2O$. In the former, the susceptibility is finite at $T=0$, but in the latter it is infinite. This diversity in behavior is because the Pr^{+++} ion has an even number of electrons, whereas the Nd^{+++} one has an odd number, so that the Kramers degeneracy, which I have already alluded to, persists right down to $T=0$. (This is on the assumption that exchange coupling is neglected—actually for very low temperatures this supposition is no longer warranted, and of course, one never realizes an infinite susceptibility.)

Theory for Salts of the Iron Group (*1932*).— It is in salts of the iron group that the intervention of the crystalline field manifests itself in

most striking form. Here the electric field is so powerful that it nearly quenches the orbital moment—essentially the same effect as already discussed in connection with polyatomic molecules. The possibility of such a quenching action was suggested more or less independently by Stoner, Pauling, and the writer. The reason that the crystalline field is able to obliterate most of the magnetic moment in salts of the iron group, and not in the rare earths, is that the $3d$ electrons are less cloistered from outside forces than are the $4f$. As a result of the quenching of the orbital moment, the susceptibility should approximately obey the spin-only formula (7). Figure 8 shows that in salts of the iron group, the spin-only model yields far better results than does the theory for free ions, which the corresponding Fig. 3 showed worked successfully for the rare earths. It is, however, the departures from the spin-only expression (7) which are particularly interesting and informative about the crystalline field. The quantitative theory of such deviations was first developed by Schlapp, Penney, Jordahl, and the writer, and has subsequently been refined by Krishnan, Opechowski, Polder, and others. One of the most striking successes of the crystalline field theory is its explanation of the marked differences in the anisotropy of the different ions, shown in Table I. There is no difficulty in understanding why manganous and ferric salts show very little anisotropy, for an S state has a completely centrosymmetric charge distribution except for distortion caused by interatomic forces. On the other hand, it is at first sight very surprising that the anisotropy of cobaltous salts should be about twenty times that of nickel ones, despite the fact that the cobalt and nickel ions are both in F states and adjacent in the periodic table. The answer is found in the inversion of the crystalline Stark pattern for Co^{++} as compared with Ni^{++}, even though the constant D of Eq. (10) has the same sign for both ions. One should not form the impression that this inversion is made in an *ad hoc* fashion; it is instead based on purely theoretical considerations similar to those which Goudsmit used in showing that ordinary spin-orbit multiplets become inverted when shells are more than half full. (The inversion in the present case, however, takes place every quarter
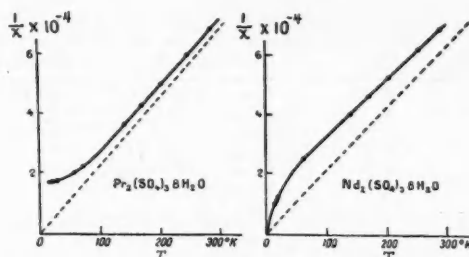
FIG. 7. Temperature variation of the reciprocal of the susceptibility per gram ion for $Pr_2(SO_4)_3 \cdot 8H_2O$ and $Nd_2(SO_4)_3 \cdot 8H_2O$.

rather than half period.) Figure 9, which is erect in Ni^{++} and inverted in Co^{++} shows in the center the decomposition of the substates of an F level by a purely cubic crystalline potential Eq. (10), and on the right the additional separations which take place when a small rhombic potential Eq. (9) is superposed. If a cubic level is nondegenerate, it is very nearly isotropic. The ground state Γ_2 of Ni^{++} is of this character. On the other hand, a single component a of a degenerate cubic level such as Γ_4 can be highly anisotropic, for the cubic symmetry is achieved only by averaging over all three components a, b, c. Hence when Fig. 9 is inverted, and when the intervals a-b-c are at least comparable with kT, making a the most populated level, there can be a high degree of anisotropy. This is the case of Co^{++}.

The discussion in the preceding paragraph has all been on the basis that the coordination number of the salt is six, the value appropriate to the hydrated sulphates so commonly used in magnetic experiments. With a coordination number of four rather than six, the constant D in Eq. (10) changes sign, and Fig. 9 is erect rather than inverted in Co^{++}. Pink and blue cobaltous compounds are characterized by coordination numbers 6 and 4, respectively. Hence blue cobaltous salts should be nearly isotropic. Krishnan and Mookherji have measured the blue tetra-coordinated salts Cs_3CoCl_5 and Cs_2CoCl_4, and find that their anisotropies amount to only $6\frac{1}{2}$ and 5 percent, to be contrasted with the 30 percent for the ordinary pink hydrated cobaltous salts. Thus, the theory predicts correctly the relation between coordination number and anisotropy. The magnetic behavior of $CuSo_4 \cdot 5H_2O$ indicates

TABLE I. Magnetic anisotropy.

Ion	State	Salt measured	Anisotropy
Mn^{++}	d^5 ^6S	$Mn(NH_4)_2(SO_4)_2 \cdot 6H_2O$	0.10%
Fe^{+++}	d^5 ^6S	$FeK_3(C_2O_4)_3 \cdot 3H_2O$	0.20
Fe^{++}	d^6 ^5D	$FeK_2(SO_4)_2 \cdot 6H_2O$	16
Co^{++}	d^7 ^4F	$Co(NH_4)_2(SO_4)_2 \cdot 6H_2O$	30
Ni^{++}	d^8 ^3F	$Ni(NH_4)_2(SO_4)_2 \cdot 6H_2O$	1.6
Cu^{++}	d^9 ^2F	$Cu(NH_4)_2(SO_4)_2 \cdot 6H_2O$	20

that the Cu^{++} in this material should have a coordination number 6, for its susceptibility behaves like to that of $Cu(NH_4)_2(SO_4)_2 \cdot 6H_2O$. For a long time chemists thought this impossible, for there are only five water molecules available per cupric ion. Later, however, the x-ray measurements of Beevers and Lipson reveal that there is actually a coordination number 6—one of the SO_4 radicals lends an oxygen in such a way that the Cu^{++} ion is surrounded by six O atoms. Thus, here the theory of magnetism actually anticipated the results of x-ray analysis in giving the type of arrangement around the cation. Krishnan and Mookherji have shown that from the x-ray data it is possible to determine theoretically the directions of the principal magnetic axes in $CuSO_4 \cdot 5H_2O$. The agreement with experiment is good.

Theory predicts that the salts of the iron group with large amounts of anisotropy should exhibit more marked departures from the spin-only formula and from Curie's law than do the nearly isotropic ones. This behavior is confirmed experimentally.

Three New Developments.—Any physical theory which does not move forward becomes stagnant and tends to become sidetracked in general interest in favor of other more rapidly moving branches of physics. From time to time I have worried lest this might happen to the theory of magnetism, but fortunately within the last ten or fifteen years there have been three striking developments which have kept it from becoming stale or hackneyed. Although superficially very different, all three have one point in common: they are all concerned with very small energy splitting of the spin levels, usually of the order of a fraction of a wave number, to be contrasted with the much larger orbital splittings, $\sim 10^3 - 10^5$ cm^{-1}, shown in Fig. 9.

Adiabatic Demagnetization (*1933– *).—The first of the three new fields to be opened is that of adiabatic demagnetization, for which Giauque has recently been awarded the Nobel Prize in chemistry. As you know, the basic idea is to cool a material by magnetizing it isothermally and demagnetizing it adiabatically, a process in some ways analagous to the cooling of a gas by isothermal compression and adiabatic expansion. The first successful experiments on adiabatic demagnetization were performed in 1933, although the theoretical possibility of such an effect had been predicted a few years earlier independently by Debye and Giauque. The average person probably thinks of adiabatic demagnetization as of interest mainly because it produces a record low in temperature. This accomplishment, considered just as such, is primarily a stunt. More fundamentally, adiabatic demagnetization is of importance as a gold mine of information on small Stark splittings and coupling energies, and on relaxation behavior at low temperatures. In all of these questions the theory of magnetism is inextricably involved. Furthermore, the theory of magnetism is useful in interpreting how low the temperature really is. As a first approximation the temperature may be determined by measuring in a weak field the susceptibility χ after the material has been demagnetized, and assuming the validity of Curie's law $\chi = C/T$. Then if we know the Curie constant C from experiments at high, i.e., helium temperatures, we can find T. However, any determination of the temperature made in this fashion is of necessity inaccurate. Accurate validity of Curie's law implies zero specific heat, so that there is nothing to resist the cooling of the ma-
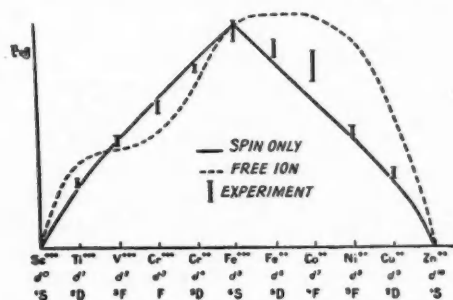

FIG. 8. Comparison of results of spin-only model and theory for free ions with experimental results upon salts of the iron group.

terial right down to the absolute zero, an obvious absurdity. Actually, crystalline Stark effects, dipolar interaction and so-called superexchange coupling (perturbation of exchange energy by excited states) give rise to specific heats and to departures from Curie's law. The first serious attempt to correct the Curie temperature scale theoretically was made by Hebb and Purcell in 1937. Since that time considerable progress has been made in direct experimental determinations of the absolute temperature by measuring the ratio dQ/dS of the differentials of heat and entropy. Such experiments are difficult, but when available are definitive and check whether the theoretical physicist has really been using the correct model for the specific heat. One particularly interesting phenomenon connected with adiabatic demagnetization is the discovery that because of either dipolar or superexchange interaction, many salts of high magnetic dilution have antiferromagnetic or ferromagnetic Curie points at temperatures about a hundred-thousandth of the Curie temperatures of ordinary concentrated ferromagnetic materials!

Paramagnetic Relaxation (1936–).—The second great impetus given to the theory of magnetism in recent years is found in the experiments on paramagnetic relaxation made by Gorter and other Dutch physicists beginning in 1936. In these experiments magnetic absorption and dispersion were detected at radiofrequencies at liquid air temperatures, and at ordinary alternating-current frequencies at helium temperatures, where the relaxation is slower. In this fashion the dispersive region connected with the transition from the isothermal to the adiabatic susceptibility may be studied, and inferences made regarding the spin-lattice and in some cases the spin-spin relaxation times. Furthermore, the specific heat of the spin system can also be deduced. It is, in fact, quite remarkable the wealth of information which is furnished by the Dutch relaxation experiments with comparatively simple apparatus.

Microwave Magnetic Resonance Spectra (1946–). —The final advance which I shall discuss is the advent of microwave spectroscopy. As a by-product of the tremendous progress in microwave techniques made during the war in con-
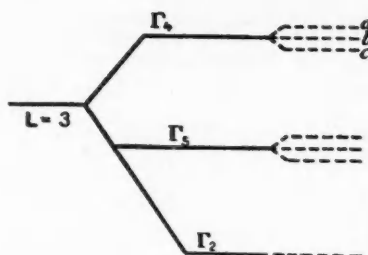
FIG. 9. The center of the figure gives the decomposition of an F oribital state ($L=3$) in a crystalline field of cubic symmetry. The right side shows the additional decomposition which results when small deviations from cubic symmetry completely remove the orbital degeneracy.

nection with radar, it is possible to measure directly energy separations amounting to a fraction of a wave number. These are intervals of just the order of magnitude needed in studying the Zeeman and Stark decompositions involved at low temperatures in adiabatic demagnetization, etc. By contrast, the splittings involved in nuclear magnetic resonance spectra, a subject we omit, are yet another thousand times smaller, and can be detected by radiofrequency rather than microwave spectroscopy. At the other extreme, Stark separations of several hundred wave numbers or more are of interest to the theory of magnetism at room temperatures, and can sometimes be detected by infrared or visible spectroscopy, as has been done by Spedding, but the interpretation is often difficult because vibrational fine-structure can easily be confused with that arising from the Stark effect.

Before the advent of microwave spectroscopy, the theoretical physicist was at liberty to assume in more or less *ad hoc* fashion any reasonable crystalline potential which could be devised to fit the magnetic data. The present epoch may be described as a "checking-up period," in which the experimental microspectroscopist tests whether the proper Stark splittings, etc., have been employed by the theoreticians. I wish I could say that the recent experiments, made in this country at M.I.T., Pittsburgh, and Rutgers, and especially by Bleaney and co-workers at Oxford, had completely confirmed previous theoretical calculations. It is true that the broad outlines of the theory are substantiated by the

TABLE II. Decomposition of the ^6S level of the Fe^{+++} ion in iron ammonium alum.

Authors	$\Delta\nu$	T	Method
Dijkstra, Gorter and Volger	0.135 cm^{-1}	90°K	Absorption
Teunissen and Gorter	0.125	64–90	Dispersion
Broer	0.135	77–90	Dispersion
Starr	0.130	77	Dispersion
Du Pré	0.125	3.5–4	Dispersion
Kurti and Simon	0.155	0.05–.2	Adiabatic demagnetization
Casimir, de Haas and de Klerk	0.120	0.27–1.2	Adiabatic demagnetization

Recent microwave measurements, however, reveal no decomposition, showing that $\Delta\nu$ must be less than 0.10 cm^{-1}.

TABLE III. Splittings (in.$^{-1}$) in various chromic alums at low temperatures.

Temperature	Ammonium	Potassium	Rubidium	Cesium	Methylamine
290°K*	0.135	0.12	0.165	0.145	0.165
193°K	0.085	0.055	0.126	0.134	
90°K	0.035 transition point	{0.26 {0.15±.01	0.108±.002	0.133±.002	0.170±.003
80°K	{0.314±.003 {0.242±.003				
20°K	{0.317±.003 {0.240±.003	{0.270±.003 {0.15 ±.01	0.108±.002	0.133±.002	0.170±.003

microwave measurements, but the data reveal that the details are often somewhat different from previously supposed, and unfortunately sometimes considerably more complicated. In iron ammonium alum, for instance, we thought that we knew the splitting quite well, for there was excellent agreement between a number of values, variously deduced, from paramagnetic relaxation and adiabatic demagnetization, as shown in Table II. The microwave experiments, however, show unequivocally that the separation must be considerably smaller than supposed. The explanation of the discrepancy is probably that the earlier determinations of the splittings are all indirect ones from the magnetic specific heat, and that part of the latter which was originally attributed to the Stark effect arises instead from superexchange coupling, which is thus shown to be more important than previously supposed.

The situation in the chromic alums is particularly striking. Table III, taken from one of Bleaney's papers, shows that here the splitting is very sensitive to the precise type of alum used, and depends drastically upon temperature, instead of being more or less independent of temperature, as theoreticians had assumed. The large change in the separation at the polymorphic transition point is not unreasonable, and the other variations with temperature can be accepted on the ground that the splitting of the spin quartet of the chrome ion is caused entirely by the deviations from cubic symmetry, a second-order effect consequently sensitive to minor reorganizations in positions. However, it is difficult to understand why sometimes two separations are observed, as shown by the braces in Table III. The Kramers degeneracy theorem is thereby contradicted unless one assumes two kinds of ion, but any such interpretation is hard to reconcile with intensity and specific heat data. In one respect the microwave results clarify previous difficulties—since the crystalline field varies with temperature, one need not be too concerned with the fact that the Indian measurements on anisotropy at liquid air and room temperatures yield for many salts different separations than those obtained from the analysis of magnetic data at low temperatures.

So far I have discussed paramagnetic resonance spectra. The field of ferromagnetic microwave spectroscopy is likewise interesting. Here a complication not found appreciably in the paramagnetic case is that the demagnetization corrections make the effective field different from the applied one, as Kittel has shown. For the geometry commonly used, the effective field has the value $(BH)^{\frac{1}{2}}$. The g-factors yielded by microwave measurements come out greater than 2, whereas those obtained from gyromagnetic experiments are less than 2. The g-factors involved in the two sets of measurements are not really the same conceptually, and in the first approximation their deviations from 2 should be equal and opposite. Even so, the spectroscopic g-factors are higher than can be accounted for on this basis, and here again is an interesting dilemma for the theoretical physicist.

Conclusion.—None of the three new tools, *viz.*, adiabatic demagnetization, paramagnetic relaxation, and microwave spectroscopy, have been exploited to anything like their full capacity. In particular, in the next few years the development of techniques with millimeter rather than centimeter waves should greatly broaden the region covered by microwave spectroscopy, and

make possible examination of some of the most interesting Stark splittings for the theory of magnetism. It may be that sometime in the distant future the theory will reach the stage of diminishing returns where advances can be made only with million-dollar computing machines. At present, however, a slide rule, even of the ten cent store variety, is adequate equipment. The next few years promise, indeed, to be far from dull for the theory of magnetism.

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The following brief list of books and articles is suggested to the reader who wishes to study in some detail the history and the present status of the theory of magnetism. References to the classical literature, which we omit documenting here, will be found therein.

E. C. Stoner, *Magnetism and Matter* (Methuen and Company, London, 1934), 575 pp.

J. H. Van Vleck, *The Theory of Electric and Magnetic Susceptibilities* (Oxford University Press, London, 1932), 379 pp.

J. H. Van Vleck, "Quelques Aspects de la Théorie du Magnetisme," *Ann. Inst. Henri Poincaré* 10, 57–190 (1948).

H. B. G. Casimir, *Magnetism and Very Low Temperatures* (Cambridge University Press, London, 1940), 93 pp.

C. J. Gorter, *Paramagnetic Relaxation* (Elsevier Publishing Company, Amsterdam and New York, 1947), 127 pp.

C. Kittel, "Physical Theory of Ferromagnetic Domains," *Rev. Mod. Physics* 21, 541–583 (1949).

L. Néel, "Propriétés Magnétiques des Ferrites, Ferromagnetisme et Antiferromagnetisme," *Ann. de Physique* 3, Série 12, 137–198 (1948).

D. M. S. Bagguley, B. Bleaney, J. H. E. Griffiths, R. P. Penrose, and B. I. Plumpton, "Paramagnetic Resonance in Salts of the Iron Group," *Proc. Physical Soc. London* 61, 542–561 (1948).

The Dimensional Structure of the Electromagnetic Field

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THE Maxwell field equations in their usual form are not well adapted to any simple dimensional analysis, and the variety of possible interpretations has caused considerable confusion. The application of these equations to the practical electrical units, as developed in the mks system, introduces field constants which are complicated and impractical. The difficulty stems directly from an unfortunate choice of primary quantities. It is the purpose of this article to show that a simple and straightforward solution of the problem is possible in terms of a dimensional structure incorporating suitable electrical primary units.

The physical concepts of length and time are so fundamental that we cannot express them in terms of anything more basic. The concept of mass lies in a different category. Its suitability as a third primary quantity rests on practical considerations. For general physical purposes the choice appears ideal, but not for the purposes of electromagnetic theory. The connection between our electrical units and the unit of mass is quite indirect, although a system of units can be derived from the traditional three primary units if either the permittivity or the permeability of space is assigned arbitrarily. Thus we have the historical situation of two cgs systems with contradictory interpretations, an electrostatic system with permittivity unity, and an electromagnetic system with permeability unity. We will not use the permittivity and permeability of space here. These two quantities have no direct significance as constants of nature, since they serve only to define the units used and the units are not defined independently of the constants.

The only constant of direct physical significance here is the field velocity c. In addition, there may be ratio factors relating units derived in different ways, factors which remain a matter of definition and choice. The "field impedance constant" is such a ratio factor. It may not have the aspects of a "dimension" from the viewpoint of fundamental theory, but from the practical viewpoint we will find it expedient to assign an independent primary unit to it, separating it from the more fundamental quantities. This much was attempted in the mkos system,[1] which retained mass as a primary quantity. We will

here proceed a step further, omitting mass from our dimensional structure and substituting a quantity more directly applicable to field theory, namely potential. The resultant structure is particularly adapted to the practical system, with the volt and the ohm serving as primary units.

Potentials in the more general sense may be regarded as the fundamental concept of field theory. When we have devised rules for determining potential functions we are in a position to answer any useful questions concerning the behavior of our field. Potentials in general may be scalar or vector quantities, may be internal or external, may be associated with the thing acting or that acted upon. Thus we have external field potentials, internal or "source" potentials associated with the regions of origin of our fields, and similar potentials associated with the entities acted upon. Current is an internal vector potential. Both the volt and the ampere may serve as potential units. Combinations of these potentials with space and time coordinates form the various quantities appearing in our equations: webers, coulombs, volts per meter, etc.

It is a familiar procedure to split a physical system into the portion acted upon and its surroundings. In the electromagnetic field this is done in a fundamentally different way. Here the usual physical quantities themselves are split to form electromagnetic quantities. The combination or interaction of two electromagnetic quantities gives us the familiar physical quantities, power, energy, force. No more fundamental formulation than this is available to us; no "field constant" enters into this relationship. The relationship volts×amperes = watts links our practical electrical units to the mks physical units. We can start with electromagnetic quantities as primary and build up a simple dimensional structure; reversing the process inevitably introduces fractional exponents.

From the theoretical viewpoint it would of course be desirable to build a system of units in terms of three primary units, corresponding to the three fundamental quantities, potential, length, and time. Such systems are possible, but those in common use contain two distinct potential units. In practice the use of resistance standards gives us a simple means of relating

the two units, with an applied scalar potential or emf and an internal vector potential or current each expressed in its own units. The two physical distinctions involved here, the distinction between a scalar and a vector quantity, and the distinction between an internal and a field quantity, are not necessarily regarded as dimensional. It is the usefulness of the concept of resistance in specifying our units which justifies us in assigning to it the status of a dimension in a limited sense. This status reduces to a simple numeric, the field impedance constant, in the structure of the field equations themselves. Concepts of this nature, which may be taken as dimensional or dimensionless at our convenience, are perhaps more common than is generally realized, a simple example being that of an angle.[2]

This concept of resistance forms the crux of our dimensional problem and we gain several advantages by retaining its independence. The four-dimensional VRLT system reduces directly to a three-dimensional VLT system if we take R as dimensionless, a useful approach for fundamental theory. However, we may also transform to other dimensional interpretations by arbitrary assignment of dimensions to R, in particular the traditional electrostatic and electromagnetic bases. In forming such systems we set the square of one potential equal to force. Since we also have the dimensional relationship $V \times (V/R) =$ power, in all consistent systems of units, we may reconcile these two relations by assigning to R the same dimensions as the ratio between force and power. Thus we have:

Electrostatic base:
$$R = T/L; \quad V^2 = (Q/L)^2 = \text{force};$$

Electromagnetic base:
$$R = L/T; \quad (V/R)^2 = I^2 = \text{force}.$$

These assignments are quite legitimate if their nature is clearly understood; each serves to simplify the description of a specific group of phenomena. Dimensions in terms of VLT are readily converted to other forms, QLT, ILT, or even MLT, by a routine process of algebraic substitution.

Dimensional interpretation is to a considerable extent distinct from the matter of units; it

[2] P. Moon and D. E. Spencer, *Am. J. Physics* **17**, 171 (1949).

is usually possible to apply more than one interpretation to each system of units, adapting specific systems to the general forms. For example, resistance in the Gaussian system may be taken as an independent dimension, may be regarded as dimensionless (the ratio of two potential units), or may be assigned the dimensions T/L to form an electrostatic base. Three-dimensional electrostatic and electromagnetic base systems have been discussed in detail in the *Journal*, and will not be considered further here.[3]

The practical unit of potential, the volt, is usually defined as a scalar function only. The ampere is not ordinarily regarded as a potential function, yet it is just that, a vector potential associated with the state of motion of the electrons in a conductor. The ampere serves as a field unit of vector potential in the expressions, amperes or ampere-turns per unit length, commonly used as magnetic field **H** units. It is a fairly simple matter to extend the meaning of these units and express all potentials, scalar or vector, in volts; or alternatively, to express all potentials, scalar and vector, in amperes. The ratio of any potential expressed in volts to the same potential expressed in amperes constitutes our "field impedance" constant or ratio. The specification that this ratio shall be numerically equal to the field velocity in meters per 10^{-7} second, serves to define the ohm of the practical system. There is nothing that requires us to assign to the ohm the dimensions of a velocity; its essence lies in an arbitrarily defined ratio factor.

Use of the volt and the ohm as primary units is well justified from the practical viewpoint. Standards of these quantities serve as primary in much practical laboratory work. These units can, in fact, be specified to a greater degree of precision in terms of such standards than they can in terms of the mks definitions. We acquire freedom to choose space and time coordinates to suit our convenience. We may combine the volt, ohm, ampere, and watt with time in seconds to define the weber, henry, farad, coulomb, and joule; or we may choose some decimal subdivision of the second for our time coordinate and reduce units of this group proportionately. We have not specified a unit of length, and as we

[3] F. W. Warburton, *Am. J. Physics* **16**, 435 (1948).

shall see we can leave this unit completely arbitrary. Complete systems of units are readily set up for any particular choice of space and time coordinates.

We will outline briefly the relations between the principal units for stationary or quasi-stationary states, before we proceed to the general field equations.

The vector potential at any point is given by the customary integral

$$\int \frac{\mathbf{I}dl}{r},$$

or a similar volume integral in terms of current density. The unit of length may be arbitrary. With I expressed in amperes the vector potential will also be in amperes. No dimensional distinction exists here between the current and its vector potential. In terms of moving charges we make the equivalent summation

$$\sum Q\mathbf{v}/r,$$

an expression again independent of the unit of length, reducing to charge/time, or amperes.

We may define a scalar potential in amperes similarly by substituting the velocity c, which appears here as an intrinsic velocity associated with the charge Q:

$$\text{Scalar potential (amperes)} = \sum Qc/r.$$

The products $Q\mathbf{v}$ and Qc have the dimensions (amperes×length) and serve to express electrostatic as well as electromagnetic effects in terms of an equivalent current, greatly simplifying our dimensional structure. Charge appears here as an active, dynamic thing, with its electrostatic effect related to the intrinsic velocity c just as its electromagnetic effect is related to the velocity v.

To express scalar and vector potentials in volts we multiply the value in amperes by the field impedance constant k, thus:

$$\text{Scalar potential } V \text{ (volts)} = k \sum Qc/r;$$
$$\text{Vector potential } \mathbf{A} \text{ (volts)} = k \sum Q\mathbf{v}/r,$$

where $k = 30$ ohms (the factor 4π does not enter here).

We may now obtain our electric and magnetic fields by taking the space derivatives of these

TABLE I. Table of dimensions.

Quantity	Dimensions Potential base	Mass-charge base	Practical unit	Erg-cm unit
Potential (scalar or vector)	V	ML^2/QT^2	volt	esu, 300 volts
Impedance, resistance	R	ML^2/Q^2T	ohm	30 ohms
Current	V/R	Q/T	ampere	emu, 10 amperes
Power	V^2/R	ML^2/T^3	watt	erg/cmt, 3000 watts
Time	T	T	second	centimeter of time (cmt)
Energy	V^2T/R	ML^2/T^2	joule	erg
Charge, electrostatic flux	VT/R	Q	coulomb	esu
Capacitance	T/R	Q^2T^2/ML^2	farad	esu
Magnetic flux	VT	ML^2/QT	weber	emu, 10^{-8} weber
Inductance	TR	ML^2/Q^2	henry	emu
Length	L	L	arbitrary	centimeter
Electric field **E**	V/L	ML/QT^2	volt per unit length	esu, 300 volts/cm
Magnetic field **B**	V/L	ML/QT^2	volt per unit length	emu (Gauss), 300 volts/cm
Force	V^2T/RL	ML/T^2	joule per unit length	dyne
Mass	V^2T^2/RL^2	M	c^2 joules	erg

potentials. Using the conventional signs we have:

$$\text{Electric field } \mathbf{E} = -\nabla V$$
$$\text{Magnetic field } \mathbf{B} = \nabla \times \mathbf{A}.$$

Both fields here appear in volts per unit length. The distinction between the two lies in the different physical patterns formed by these two derivatives. The derivative of the scalar potential is a simple vector, while the curl of the vector potential is actually a tensor of the second rank.[4] The representation of this quantity by means of a vector is analogous to the representation of an angular momentum or velocity by means of a vector along its axis.

There is an advantage in expressing both fields in the same units, as this serves to distinguish essential physical differences from the man-made factor of units. We have available a choice of units for our field quantities and the choice is largely a matter of convenience. Thus we can express the fields in terms of potential (amperes or volts) per unit length; or we can divide these quantities by the field velocity c to give the "flux density" units, coulombs or webers per unit area. It is desirable that the dimensional relations between these units be kept explicit and obvious.

In the basic differential equations we shall use the products $(\rho\mathbf{v})$ and (ρc) to express the

[4] A. Einstein, *The meaning of relativity* (Princeton University Press, 1945), p. 22. The tensor nature of the magnetic field is here discussed.

source quantities as an ampere density, multiplying by k to convert all potentials into volts. Writing 4π explicitly in these equations avoids its appearance in the integral forms. The notation $T = ct$ converts time into units of length. The unit of length remains completely arbitrary, since force may be defined as energy (joules) per unit length, and the fields are expressed in volts per unit length.

At this point we could assume the usual Maxwell field equations and simply substitute the constants k and c. We shall not follow this procedure here. There is substantial evidence that the field velocity c represents something more basic than the field equations themselves. We find this velocity associated with radiation in general, and with the structure of mass itself. The constant c appears in the Lorentz transformation, and in the equation energy $=mc^2$, relations which certainly do not depend on the usual field equations. The ability of space to contain and transmit energy with the velocity c appears as a fundamental physical fact. We will here reverse the usual order, and derive the electromagnetic field equations for free space from the following postulate: "It is possible to describe the electromagnetic field in terms of potential functions which satisfy equations of propagation and continuity; resultant forces are proportional to the derivatives of these potentials." We may then modify these conditions as necessary to take care of additional factors in regions occupied by matter.

The fundamental differential equations are:

Propagation:

$$(\nabla^2 - \partial^2/\partial T^2) V = -4\pi k(\rho c)$$
$$(\nabla^2 - \partial^2/\partial T^2)\mathbf{A} = -4\pi k(\rho \mathbf{v}).$$

Continuity:

$$\nabla \cdot \mathbf{A} = -\partial V/\partial T.$$

Force:

$$f = Q[\mathbf{E} + (\mathbf{v}/c) \times \mathbf{B}],$$

where $\mathbf{E} = -\nabla V - \partial \mathbf{A}/\partial T$.

We derive:

$$\nabla \cdot \mathbf{E} = 4\pi k(\rho c) \qquad \nabla \times \mathbf{E} = -(\partial/\partial T)(\nabla \times \mathbf{A})$$
$$\nabla \cdot \mathbf{B} = \nabla \cdot (\nabla \times \mathbf{A}) = 0 \qquad \nabla \times \mathbf{B} = (\partial \mathbf{E}/\partial T) + 4\pi k(\rho \mathbf{v}).$$

The dimensional structure of these equations is quite simple. The force equation has the dimen-

sions (amperes×time)×(volts/length), equal to joules/length, or force; if we multiply both sides by c this becomes simply watts = amperes×volts. The other equations describe the mode of distribution of the potentials and contain in essence only two quantities, potential and length. The products (ρv) and (ρc) are functions of the internal or "source" potentials, with the dimensions amperes/(length)². The unit of time cancels in these products, and time appears elsewhere in the equations already converted to units of length.

In extending our equations to regions occupied by matter no additional problem of units or dimensions is involved. It is frequently desirable to distinguish the effect of "concealed" charges and currents from the obvious sources, and it is for this reason that we are justified in introducing the quantities **D** and **H**; these quantities occupy a subordinate role in fundamental theory. The dimensional problem here is an artificial and illusory one. It is always possible to express **B** and **H** in the same units, giving a purely dimensionless relation between them which reduces to an identity in free space; similarly for **E** and **D**. We have available the same choice of units for **H** and **D** that we have for **B** and **E**. These quantities all have the same dimensions if they are expressed in the same units; dimensional differences are associated with differences in choice of units. The confusion so frequently surrounding these quantities has its origin in the failure to keep separate the dimensional and the dimensionless relations between them.

It may be well to illustrate these relations by means of a simple numerical example. Given an electromagnet of one square inch cross section with an air gap 0.2 inch long, magnetized by 200 ampere-turns. Neglecting minor factors, the field in the air gap is 200 ampere-turns/0.2 inch = 1000 ampere-turns per inch. We can express this in the following alternative forms:

Multiplying by 4π gives us 12,560 amperes per inch.

Multiplying this by k gives 376,700 volts per inch. This is the curl of the vector potential in volts.

Dividing this by $c(=1.18\times10^{10}$ inches per second) gives us 3.19×10^{-5} webers (volt-seconds) per square inch.

We can also divide the amperes per inch by c to give 1.065×10^{-6} coulombs per square inch, where the coulomb is used as a *magnetic* flux unit. Multiplying by k then gives us webers per square inch, as before. By substituting microseconds in the value of c we may obtain microwebers and microcoulombs directly.

In the iron-core portion of our field we can use any of the above units for **B**, dividing by the relative permeability to express **H** in the *same* units. Such an overabundance of units may not be needed in practice; still an understanding of the relations between them is of basic importance.

The position and relationship of our field quantities can perhaps be further clarified by specific consideration of the energy equation. Energy density in free space is given by $(\mathbf{E}^2+\mathbf{B}^2)/8\pi ck$, where **E** and **B** are in volts per unit length. In this expression the division by k changes (volts)² into watts, while the division by c converts watts/(length)² into energy per unit volume. If we choose amperes instead of volts for **E** and **B** the factor k will be transferred to the numerator, thus: $(\mathbf{E}^2+\mathbf{B}^2)k/8\pi c$. The arbitrary nature and position of k is here evident, as is the dimensional identity of **E** and **B**. These expressions exhibit directly the corresponding units for the two potentials and their derivatives, a correspondence which occurs initially in the equation of continuity.

The energy equation is readily extended to regions occupied by matter if we write it in the form $(\mathbf{ED}+\mathbf{BH})/8\pi ck$, expressing all four quantities in volts per unit length. However, if we introduce the relative values of the permittivity ϵ and the permeability μ, retaining the field vectors **E** and **B**, we obtain the form

$$(\epsilon\mathbf{E}^2+\mathbf{B}^2/\mu)/8\pi ck.$$

It is customary to substitute **H** for **B** at this point, transferring μ to the numerator. This procedure is comparatively harmless as long as we use relative values of ϵ and μ. However, when these quantities are combined with dimensional factors to form complicated "field constants," requiring different units for **E** and **H**, as is done in the usual version of the mks system, the result attains the ultimate in confusion. The inverse position of ϵ and μ is inherent in their definitions, as polarization subtracts in one case

and adds in the other. This fact may well be recognized explicitly. The essential point of similarity between electric and magnetic fields is a dimensional correspondence between two different physical structures; the units of each can be applied to the other.

We have here expressed our equations in the "practical" system of units; it is the most familiar and it offers a sufficiently wide choice of units to illustrate the dimensional relations involved. The dimensional formulation outlined may also be applied to units of the cgs systems. For the Gaussian system we set $k = 1/c$. This may be taken as a numerical relation only, as the dimensional and physical significance of k is entirely different from $1/c$. Cancelling these two constants leaves an incomplete picture.

A simpler system is obtained by setting $k = c = 1$, expressing time as well as length in centimeters. In this system k and its corresponding dimension may be omitted at the outset, but it is generally desirable to distinguish space and time coordinates. Electrostatic quantities appear in electrostatic units, while electromagnetic quantities, including current, appear in electromagnetic units. This system is completely determined by two basic units, which we may take to be the erg and the centimeter.

What we have outlined here and summarized in Table I is not primarily a system of units, but rather an analysis of units and constants into their component parts. Mathematically we have done little more than break down the traditional field constants into their components. Yet this is an essential step in analyzing the problem, and serves to display the physical structure of the electromagnetic field in a clearer perspective. This structure reduces to one basic concept, potential. The potential functions applicable are simple scalar and vector quantities, and in free space these obey two familiar physical conditions, propagation and continuity. The first assigns to our field energy a velocity c, an attribute which appears to be shared by energy in general, including the intrinsic energy of mass and charge. The second signifies that our field energy may be taken as a continuous, conservative quantity for the purposes to which the field equations are applicable. Then the discontinuity known to exist at a smaller order of magnitude at once sets a lower limit to the validity of the equations. The equations are not adequate, for example, to account for the structure of the atom or even the electron; these involve more complex physical factors. The equations of the electromagnetic field here emerge as a comparatively simple structure with a limited range of validity. They are not to be taken as something ultimate.

Physics Today

Nearly three years ago the American Institute of Physics undertook the publication of *Physics Today*, a monthly magazine of general physics interest, as part of a program designed to develop physics and its applications to human welfare. Already the Institute is nineteen years old. Its Governing Board regards its approaching twentieth anniversary not only as a time for celebration of past accomplishments, but as an opportunity for critical evaluation of current activities.

As part of this evaluation, the role of *Physics Today* is undergoing careful consideration by a new Editorial Board recently appointed by the Executive Committee of the Institute Board. The primary objective of this new Editorial Board is to make *Physics Today* a more effective instrumentality than it has been for the unification of the membership of the Institute, and, if possible, for broadening the base of this membership by extending the community represented by the Institute into neighboring technical fields of application of physics. The second objective is to improve the financial position of *Physics Today*, and through it, the position of the Institute as a whole. Since its first appearance *Physics Today* has found itself unable to expand its subscription and advertising income sufficiently to support the type of journal it has become. Improvement of the financial position demands such immediate changes in quality of paper and in make-up as can be justified without decreasing the fundamental interest or appeal of the journal to its present readers.

An important innovation will be the establishment in *Physics Today* of a forum with leading articles of an editorial nature by members of the Editorial Board. These will express the individual opinions of Board members on subjects of current interest to physicists. It is hoped that this new forum will stimulate a lively and refreshing interchange of opinions on all matters that affect the well-being of the members of the profession and its supporters.

NOTES AND DISCUSSION

A Demonstration Unit for Magnetostriction

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THE magnetic phenomenon causing contraction or expansion of ferromagnetic materials in a magnetic field is known as the "Joule effect" or magnetostriction. It has been the subject of much research but has been difficult to measure or demonstrate because of the microscopic dimensional changes. Figure 1 shows a comparison
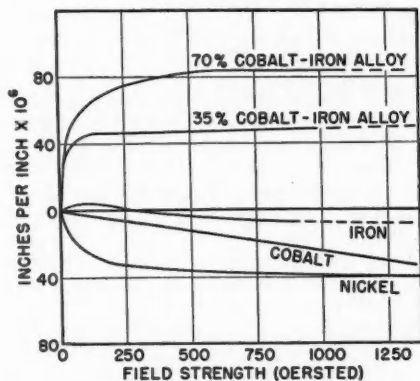
Fig. 1. Variation of magnetostriction with magnetic field strength.
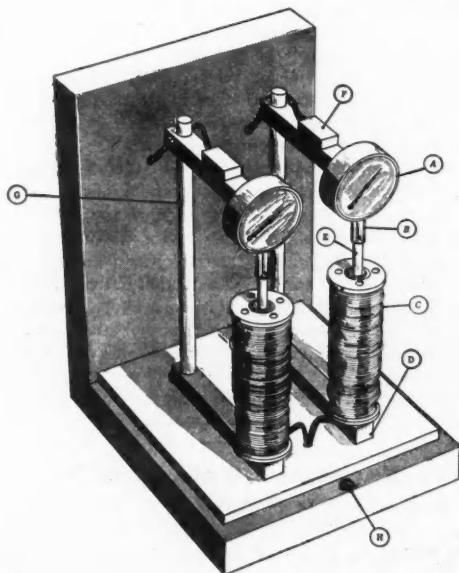
Fig. 2. Demonstration unit for magnetostriction. *A*, dial gauge; *B*, nonmagnetic spacer; *C*, solenoid; *D*, solenoid clamp; *E*, specimen; *F*, buzzer; *G*, support; *H*, switch.

of the magnetostriction of several ferromagnetic metals and alloys.[1]

Dimensional changes of highly magnetostrictive materials may now be easily demonstrated semiquantitatively by making use of modern, high-sensitivity dial gauges. Satisfactory equipment for demonstration of magnetostriction requires only a solenoid, a dial gauge having a sensitivity of 0.002 mm per division, and a simple brass stand and clamps.

Figure 2 illustrates a demonstration unit for use with two specimens. The solenoids are made up of 4000 turns of No. 21 gauge wire inductively wound on 2.54-cm o.d. Bakelite tubes 20 cm in length with 200 turns per cm. The solenoids are hooked in series and draw approximately two amperes when connected to a 110-volt d.c. line. This current produces a field of approximately 500 oersted within the solenoids in the absence of a core. The clamp stand is made of brass to eliminate magnetic interaction between the stand and specimens. It supports the gauges and solenoids and fixes their positions with respect to the magnetostrictive rods.

The metals used for demonstration are nickel, which has a negative saturation magnetostriction of 38×10^{-6}, and a 70–30 cobalt-iron alloy with the unusually high positive saturation magnetostriction of 82×10^{-6}. Both the nickel and cobalt-iron alloy rods are 1.27 cm in diameter and 30.5 cm in length. The rods have a threaded section at one end to screw into the brass base plate which fixes the samples in a vertical position. The solenoids and clamps
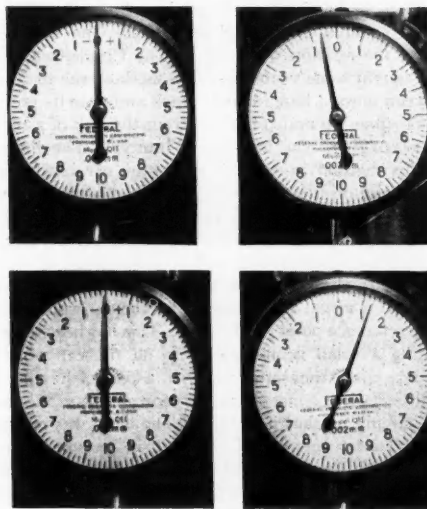
Fig. 3. Magnetostriction of nickel (above) and 70–30 cobalt-iron alloy (below) as indicated by dial gauges. Photographs at the left give the reading with the magnetic field off; those at the right with the field on.

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are slipped over the specimens and clamped into position and then the gauges are supported over nonmagnetic spacers which are fitted to the free ends of the rods. Small buzzers mounted on the gauge clamps overcome the sluggish action of the gauges and stabilize the zero reading.

Figure 3 shows the dial positions when the magnetic field is on and off. The shrinkage of the nickel is 0.006 mm or 3 dial divisions, and the expansion of the cobalt-iron alloy is approximately 0.014 mm or 7 dial divisions. These values are less than would be expected for saturation magnetostriction, probably because the specimens do not form completed magnetic circuits. The ratio of the deflections, however, is of the same order as the deflection ratio at saturation.

For measurements of less highly magnetostrictive materials preliminary efforts to increase the gauge reading by means of a lever have been promising. The specimen acts upward on the lever against the gauge and a knife-edge. Definite arm distances along the lever, which is rectangular hollow tubing, are obtained by attaching stainless steel cones, the tips of which rest on the end of the specimen under measurement. With this device, it has been possible to increase the dial readings tenfold.

[1] These values have been taken from: Thomas Spooner, *Properties and testing of magnetic materials* (McGraw-Hill, 1927), p. 117; *Magnetostriction* (Development and Research Division, The International Nickel Co., Inc., June, 1948).

Demonstrations of Acceleration

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IN demonstrating the motion of a body with constant acceleration to a large group of students it is often difficult to make clearly evident the relation between the distance covered and the elapsed time. One method of showing the relation was used at The Citadel where the writer taught some years ago. This method was to allow a car to run down a long inclined track and note its position at equal time intervals obtained from the tick of a metronome set to give about half-second intervals. The difficulty with this method is twofold. First, the car itself makes enough noise on the track to obscure partially the metronome ticks; second, there is a difficulty to the students in coordinating the visual position of the rapidly moving car with the auditory signal from the metronome.

In order to eliminate these difficulties it occurred to the writer that a visual signal carried by the car would give a definite reference point on the track. This has been done by mounting a small neon glow lamp on the car which is flashed at equal time intervals with a pair of hearing aid 45-volt batteries connected to the neon glow lamp through an RC circuit, as shown in Fig. 1. The lamp was removed from its base to get rid of the built-in resistor, and remounted with the RC circuit and batteries as a complete package on the car.

The values of the resistance and condenser were chosen to flash the lamp at intervals between one-half and one second. The nominal values used were $R=0.75$ megohm and $C=0.5$ mf. The time interval is proportional to RC.
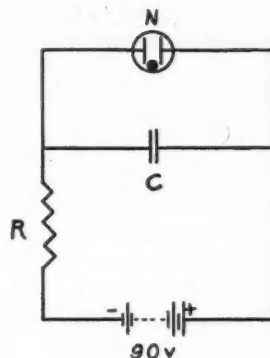
FIG. 1. Circuit for flashing neon lamp.

However, if C is chosen too small, the energy storage is small and the flashes weak; if C is too large and R made small, the lamp does not extinguish between flashes and simply flickers. The lower limit of the resistance and upper limit of the capacity used for a given flash interval will depend upon the type of glow lamp used. The neon lamp is shown at N.

Reference chalk marks were placed along the side of the track at half-meter intervals and the track elevated at one end. A release catch could be triggered at the instant of a flash numbered zero, and the position of successive flashes noted by the class as the car ran down the track. These points were then marked and their distances measured from the starting line. The position of the small lamp could be estimated to within about 2 cm for the third and last usable flash, with the constants used. The time interval used was about 0.7 sec and the length of track about 4.5 m. With an elevation of the end of the track of about one meter there was time for three complete time intervals during constant acceleration of the car. An average of three successive trials resulted in confirmation of the equation $s=\frac{1}{2}at^2$ to within about two percent.

When a metronome was used with this demonstration there was usually an uncertainty in the position of the car on the track of about ±10 cm even after several trials. Using the neon lamp as a timing device, the uncertainty of the position of the lamp on the track was reduced to about ±2 cm on the first trial. This reduction in uncertainty resulted in a marked increase in interest and confidence, on the part of the students, in the results obtained. It is felt that this method is a definite improvement over the use of an auditory signal.

Note on the Path of a Projectile

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WHEN the motion of projectiles is being discussed in an elementary physics class it is usually possible to arouse some interest, especially among the better students, by mentioning the peculiar effects produced by the Coriolis

acceleration, even if it is not feasible to include in the discussion many mathematical details.

Recently, when one of us (RTE) was carrying on such a discussion with an elementary class, the suggestion was made that, because of the Coriolis acceleration, it might not be possible to fire a projectile in a nearly vertical direction in such a way that it would return to its starting point. The argument, as presented in class, was along the following lines. If the projectile is fired "straight up" (parallel to a plumb bob) it would return to a point slightly to the west of the starting point.[1] This slight westward motion would give rise to a secondary effect causing the projectile to land very slightly north of the starting point. The path of the projectile is actually a skew curve, and the skew curve is such that altering its projection on the north-south plane has an effect on the projection on the east-west plane, and *vice versa*. The problem becomes more difficult to analyze, of course, when the viscosity of the air and the effect of winds are considered. When these latter effects are included it is by no means intuitively obvious that one can adjust the original firing angle in such a way as to make the skew curve intersect itself at the origin.

One of the brighter members of the class had his interest aroused enough to raise the question in his mathematics class. It was suggested by the instructor, one of the authors of this note (IMN), that this is the kind of problem that can be attacked by the methods of topology. The proof that the skew curve can be made to close on itself at the origin is simple and proceeds as follows.

Consider the trajectories of projectiles fired in various directions from a fixed point P with constant initial speed. The direction of fire can be indicated by the angular members of a spherical coordinate system, say angle ϕ with the vertical, and angle θ as the direction angle of the projection

of the initial velocity onto the plane. We impose the restriction $0 \leqslant \phi \leqslant 30°$ for convenience. For each pair of values (θ, ϕ) there is a unique endpoint Q of the trajectory, with coordinates (x, y) in the plane. The fixed point P may be considered as $(0, 0)$, the origin of the (x, y) system.

As the values (θ, ϕ) vary continuously, so do (x, y). We prove that there is at least one pair of values (θ, ϕ) for which the corresponding endpoint Q is the point P, $(0, 0)$.

To prove this, we first consider the values $(\theta, 30°)$: as θ varies continuously from 0 to 360°, the corresponding points Q will range over a continuous curve which is closed. This curve will wind completely around the point P under any reasonable wind conditions. Let us designate this curve by $C(30°)$, so that $C(\phi)$ in general will represent the curve of points Q corresponding to a fixed angle ϕ and a varying angle θ, $0 \leqslant \theta \leqslant 360°$. Thus $C(0)$ is a single point, say P_0, and as θ varies continuously from 30° to 0, the curve $C(\phi)$ is deformed continuously from $C(30°)$ to P_0. But a curve which winds around a fixed point P cannot be continuously deformed into a point P_0 without passing through the point P at some stage of the deformation. This assertion, which is evident from a geometrical viewpoint, is a theorem in plane topology.[2] This means, then, that at some stage of the deformation there is a direction (θ, ϕ) for which the corresponding Q point is $(0, 0)$.

This actually proves that there is a direction of fire which will have any prescribed point inside the curve $C(30°)$ as endpoint Q. No assumptions need be made about neglecting air resistance or wind drift except the reasonable one that the velocity given to the projectile by wind drift shall not exceed the muzzle velocity of the projectile.

[1] Page. *Introduction to theoretical physics* (Van Nostrand, 1935), 2nd ed., pp. 97–108.
[2] P. Alexandroff and H. Hopf, *Topologie* (Berlin, 1935), p. 463.

Proceedings of the American Association of Physics Teachers

The Middletown Meeting, June 20–22, 1950

THE June meeting of the AAPT at Middletown, Connecticut, was one of the most stimulating and enjoyable ever sponsored by the Association. This was in no small measure due to the cordial hospitality with which Wesleyan University through its president, VICTOR L. BUTTERFIELD, and the staff of the Department of Physics received our members and guests. The local committee had so carefully planned every detail of registration, lodging, meals, entertainment and program facilities that not a single complaint was heard during the three-day session. The weather was ideal save for a local shower which caught the early risers returning from breakfast Thursday morning. A spirit of earnestness mingled with good will and informality was current among the group throughout the entire period. Wesleyan University, a firm believer in the natural sciences since its founding in 1831, was a perfect host in every way.

To this setting the program chairman, VICE-PRESIDENT MARK ZEMANSKY, and his associates brought a well-balanced series of symposia and invited talks as well as a fine group of short contributed papers. The program committee

must accept no small credit for the success of the meeting.

Tuesday morning was spent in registration, getting oriented, and renewing acquaintanceships while visiting the physics laboratories. The latter were all in perfect order and adequately manned by staff members and well-informed alert assistants ready to answer questions and to direct visitors. The spirit of good teaching was kindled and quickened by the inspection of the layouts for student experiments and by the research work in progress. Even a casual survey of the laboratory gave one a clue to the reason for the high quality and large number of physics majors among Wesleyan graduates.

The afternoon was devoted to a symposium on piezoelectricity. PROFESSOR W. G. CADY, pioneer and past master in this field, discussed the discovery, development, trends and applications of the phenomenon. In his opinion the subject should be treated in a basic chapter on dielectrics and not in a section on gadgets. This lecture was followed by a series of demonstrations relating the fundamental electrical, mechanical and thermal phenomena in crystals. Each of the several demonstrations was carried out by individual students. PROFESSOR K. S. VAN DYKE had general charge and gave the necessary explanations. A descriptive talk on barium titanate by HANS JAFFE, *Brush Development Company*, and a movie (by courtesy of the Bell Laboratories) on the growth of crystals concluded the formal part of the afternoon session.

A short address of welcome by PRESIDENT BUTTERFIELD, a stimulating paper on "Teaching by Publication" by PROFESSOR L. W. McKEEHAN together with a lively exchange of appropriate repartee between TOASTMASTER ROLLER and the speakers highlighted the evening banquet.

A symposium on "The Demonstration Lecture as an Art" had been arranged for Wednesday morning. The topic was fully exemplified in a splendid series of demonstrations by PROFESSOR V. E. EATON and his assistant. This performance characterized by simplicity of apparatus and explanation, on a scale which made everything clearly visible to everyone, and with a finesse

in execution, was to many the main feature of the entire meeting.

PROFESSOR E. M. ROGERS then, in a lighter vein, recounted some incidents and situations (embarrassing and otherwise) which are likely to befall a demonstration lecturer. The remainder of the morning was devoted to an exposition of the methods and effectiveness of the demonstrations of PROFESSORS SNOW, FOLEY and POHL. A former student of each of these well-known lecturers presented individual impressions and recollections. The symposium brought out very clearly that the demonstration lecture is truly an art and that the individuality of the lecturer is a most important element. Contributed papers on apparatus and techniques occupied the attention of the group Wednesday afternoon.

Four invited papers were presented Thursday morning. For the past two years Wesleyan University has had a committee making a statistical study of the factors significant in the undergraduate training of American scientists. DR. ROBERT KNAPP gave an illuminating summary of the findings. Although physicists form only a lesser portion of the cases studied, all physics teachers will want to read the complete report when it is published. Seemingly the most meaningful factors in teaching were: energy, masterfulness, enterprise, warmth and breadth of interest and professional eminence (listed in order of their importance). Other factors such as geographical location, size, endowment, etc., have also been evaluated.

PROFESSOR L. L. BARNES led a discussion on the second-year premedical course in physics. He volunteered to serve as a "clearing center" for all who wished to exchange ideas on the subject. A surprisingly large number of departments have second-year physics courses for premedical students and they promptly accepted the suggestion of exchanging information. The next paper, read by PROFESSOR THEODORE SOLLER, described in detail the organization, subject matter, and operation of the first of a two-year science program at Amherst College.

PROFESSOR T. B. BROWN, chairman of the committee on the Taylor Memorial Manual of Advanced Undergraduate Laboratory Experiments, reported on the activities and plans of the committee. A fairly definite plan for the

manual has been formulated and a search for an editor is under way.

Contributed short papers and demonstrations Thursday afternoon concluded the activities of the scheduled program. A most encouraging aspect of the contributed paper sessions was the response by way of questions and comments from the floor.

The social needs were adequately provided for by a "punch-and-cookie" tea at the Honors College Tuesday evening, a theater party drive to the Red Barn circuit at nearby Canton Wednesday evening, a boat trip down the Connecticut River for ladies and children Thursday, and by opening the swimming pool, tennis courts and the faculty club to all guests. The weather prevented the planned visit to the observatory and a beach party.

The record attendance and the over-all success of the meeting indicates clearly that AAPT in the future may plan for more meetings of this type.

LESTER I. BOCKSTAHLER
Northwestern University

Invited Papers and Reports

Symposium on Piezoelectricity

Introduction to piezoelectricity. W. G. CADY, *Wesleyan University.*

Ten demonstrations in piezoelectricity. K. S. VAN DYKE, *Wesleyan University.*

Electromechanical properties of barium titanate. HANS JAFFE, *Brush Development Company.*

Color film: Crystal clear. A. C. WALKER, *Bell Telephone Laboratories.*

Symposium: The Demonstration Lecture as an Art

Introductory remarks. R. M. SUTTON, *Haverford College.*

Demonstration lecture. V. E. EATON, *Wesleyan University.*

Lecture experiences. E. M. ROGERS, *Princeton University.*

The lectures of the late Professor Benjamin Snow. J. C. BLANKENAGEL, *Department of German, Wesleyan University.*

What remains twenty years after a demonstration. L. I. BOCKSTAHLER, *Northwestern University.*

The lectures of Professor Robert Pohl of Göttingen. E. W. CASPARI, *Department of Biology, Wesleyan University.*

Invited Papers

Statistical study of undergraduate training of American scientists. ROBERT KNAPP, *Department of Psychology, Wesleyan University.*

Second-year physics course for biology and premedical students. L. L. BARNES, *Cornell University.*

First year of two-year science program at Amherst College. THEODORE SOLLER, *Amherst College.*

Committee Report

Taylor Memorial Manual of Advanced Undergraduate Laboratory Experiments. T. B. BROWN, *George Washington University.*

Contributed Papers, with Abstracts

Two sessions were devoted to the following contributed papers:

1. A laboratory examination for general college physics. W. H. KINSEY AND R. A. RHODES II, *University of Connecticut.*—The general reasons why examinations are given have been applied to the elementary physics laboratory at the University of Connecticut. The result is a laboratory final examination, presented at the end of each semester, in which the student, working alone, can show, both in writing and by practical demonstration, his acquired knowledge and ability in laboratory procedure and technique. Particular reasons for the existence of this examination are that (a) the students, with the final examination in sight, will take the work more seriously, looking upon it as methods to be understood and remembered rather than mere cookbook exercises to be forgotten immediately, (b) although they work in pairs during the semester, each one will learn how the apparatus operates and not let his partner do all the work and (c) the results of the final will divide the students into three levels: (1) those who are good and have the ability to think, (2) those who are simply good and (3) those who are poor. These examinations have been given at the University of Connecticut for the past 15 years, the results have been analyzed on the basis of the reasons set forth above, and the program judged to be an effective one.

2. A student spectrometer from surplus equipment. RALPH A. LORING, *University of Louisville.*—Spectrometers have been designed and built using the circle from war-surplus alidades. Certain changes from the standard spectrometer have been made: (1) The slit is simple and non-adjustable. (2) No telescope or collimator tube is used. (3) The telescope and collimator lenses are locked in. (4) All focusing is done at the eyepiece of the telescope. (5) Leveling screws are omitted from the prism table. (6) A vernier with least count of one-tenth degree is used.

3. Demonstration experiments in electromagnetic induction. D. S. AINSLIE, *University of Toronto.*—The paper described experiments designed to illustrate the principles of electromagnetic induction, which are extensions of those ordinarily given in elementary textbooks on physics. Arago's experiment, using a rotating copper disk and a

magnet, was modified by substituting an electromagnet for the permanent magnet. The use of laminations to reduce eddy currents was illustrated by rotating a laminated copper cube in a magnetic field. For one direction of rotation with the laminations parallel to the field there was very little damping compared with a marked effect when the laminations were perpendicular to the field. For the a.c. demonstration set, a cylindrical copper ring consisting of two concentric sections each with a slot was employed. The rings may then be used either as a closed- or open-circuited conductor by rotating one with respect to the other. A group of experiments was described to study the action of an induction coil in which a neon lamp was employed to gauge the secondary emf.

4. Short demonstration experiments without words. ERIC M. ROGERS, *Princeton University.*

5. Elementary physics experiments for premedical students. NORA M. MOHLER AND LILLY LORENTZ, *Smith College.*—The premedical students take the regular beginning course with an additional laboratory period devoted to demonstrations and experiments designed to relate the physics topics to problems in physiology or in instrumentation. Typical subjects covered are: the back as a lever, a metabolism test to correlate oxygen consumption, heat of combustion and energy needed by the body in a state of rest or following muscular activity, the electrical resistance of a body and its fluctuations with the emotional state, individual temperature variations and pulse rate changes as a function of time of day and as affected by activities. A lecture on the physics of hearing is followed by an audiometer test. The physiology of the eye is discussed and is followed by experiments on the optics of the eye, the variation of acuity of vision with illumination, color and color vision. A lecture on the conduction of electricity by nerves and the generation of emf in various bodily activities is supplemented by the taking of an electrocardiograph as well as by study of a breadboard cathode-ray oscillograph. Our aim was to teach some physics that would seem relevant; the result seems to be that it "sparks" interest in the whole field.

6. The simple pendulum. G. PRESTON BURNS, *Mary Washington College of the University of Virginia.*—The derivations of the equation for the period of the simple pendulum (a) by equating the accelerating force on the bob to the product of the mass and linear acceleration of the bob and then applying the equation for the period of a simple harmonic motion and (b) by regarding it as a special case of the physical pendulum were discussed. It was pointed out that the first of these, used in the majority of college physics textbooks, is inadequate in that it does not make use of the radius of the bob or make clear the reason for the assumption of a point mass. The second yields the formula

$$T = 2\pi \left[\frac{l}{g} \left(\frac{2r^2}{5l^2} + 1 \right) \right]^{\frac{1}{2}} \quad (1)$$

for the period of vibration of a sphere of radius r suspended

by a string of length l. If $r = 0$, this reduces to $T = 2\pi(l/g)^{\frac{1}{2}}$. It was recommended that the simple pendulum be treated as a special case of the physical pendulum and that Eq. (1) be used for the calculation of the period of the laboratory pendulum unless it is clear for the particular case in question that the term $2r^2/5l^2$ is sufficiently small to introduce only negligible error.

7. Physical ideas, their content, logic and social contexts, in the education of humanities majors. ROBERT S. COHEN, *Yale University and Wesleyan University.*—Designed primarily for students not majoring in science, a year course for juniors and seniors is offered at Wesleyan University with several related but distinct ends in view: to give a direct acquaintance with the physical nature of the world as exemplified by various aspects of the physical sciences; to show the logical structure of physical theories as a form of knowledge; to relate the life of science as a human activity with other human activities, historically and sociologically; to demonstrate the role of science in the formulation and solution of typical philosophical questions. The fundamental ideas underlying physical science and the scientific method are developed in their historical context and applied to significant problems, both classical and contemporary, one factor in selection being the unity of science. Six texts are required, the major one in physics. During the fall term, a long paper is written, the topic being the student's choice from a list of several dozen; in the spring term, biweekly outside readings are assigned from a very large library reserve. Tests are frequent, detailed, but short. (Lists of texts, outside paper subjects and reserve reading list were available at the meeting, along with sample tests.) The course is given primarily by informal lectures, students being encouraged to interrupt; to a certain extent it is integrated with a concurrent upperclass course in intellectual history. It seems possible to combine emphasis on the "strategy and tactics of science" with teaching of the knowledge gained by these activities.

8. Graphical method for determining galvanometer characteristics. W. D. KNIGHT AND R. F. McCUNE, *Trinity College.*—A method was described for determining the current sensitivity and resistance of a d'Arsonval wall galvanometer. The galvanometer was shunted by a known variable resistance, and the combination was fed from a constant-current source of known strength. The reciprocal of the deflection was shown to be a linear function of the reciprocal of the shunt resistance. The intercept of this straight-line function gave the current sensitivity, and the slope gave the product of the sensitivity and the resistance. The accuracy of this method can be comparable to that of conventional methods and depends on the accuracy with which the strength of the constant current source can be determined, and on graphical precision. The method is not good for demonstrating the dependence of current sensitivity upon deflection. Some advantages are: only one set of readings need be taken; the circuit is simple; and students find it a good example of the use of graphs.

9. A driver for the Calthrop resonance-pendulum. PAUL F. BARTUNEK, *Lehigh University.*—J. E. Calthrop has described a very interesting experiment with a resonance pendulum.[1] Calthrop's experiment provides a study of resonance and determines the damping coefficient in the appropriate equation of motion. In brief, the apparatus consists of a heavy compound pendulum which is used to drive a much lighter approximately simple pendulum. A slight disadvantage exists in that it is necessary to keep starting the heavy driver pendulum since the amplitude dies down with time. To overcome this disadvantage a driving apparatus free from intermittent contacts has been devised. The driving apparatus makes use of the well-known principle that if an iron core is introduced into a solenoid which forms part of a series-resonant *LCR* circuit, the latter is thrown out of resonance, causing a large drop in the current. One has only to attach a short length of iron rod to the bottom of the pendulum in such a way that the iron rod passes into the solenoid at the extreme of each pendulum swing. As the rod approaches the solenoid the current is large. Hence the iron rod is pulled in. As it goes in, the current falls to a very small value with the result that the pull is reduced to such extent that the pendulum can swing back again. The pendulum thus receives a slight tug at the extreme of each swing to maintain the motion.

[1] J. E. Calthrop, *Advanced experiments in practical physics* (Wm. Heinemann Ltd., London), 1st ed., p. 14ff.

10. Introduction to polarization of electromagnetic waves. C. L. ANDREWS, *New York State College for Teachers and General Electric Research Laboratory.*—Demonstrations were made of polarization of electromagnetic waves with a hand-sized transmitter of 12-cm waves and a hand-sized intensity meter. The direction of polarization of the electric field parallel to the antenna of the transmitter was readily recognized. The intensity meter itself served as an analyzer. Polarization by a parallel wire screen, polarization by reflection from a dielectric and elliptical polarization of radiation from two secondary sources were shown.

11. Elementary lecture demonstration with microwaves. WILLIAM M. FAIRBANK, *Amherst College.*—A lecture demonstration designed for a science course taken by all freshmen at Amherst College was discussed and given in part. War surplus radar equipment was used to demonstrate both the optical properties of microwaves and radio modulation. Modulation was first illustrated by periodically interrupting the beam between the transmitting and receiving antennas and observing the periodic motion of the galvanometer. When a suitable object was moved along a line between the transmitting and receiving antennas, periodic fluctuations in the received signal occurred due to the changing standing wave pattern between this object and the receiving antenna, the frequency of modulation being proportional to the velocity of the moving object. When the receiver was connected to an audio amplifier and speaker, audio tones of controllable frequency were heard as the object was moved between the two antennas at varying speeds. Electrical modulation was then intro-

duced by applying the voltage output from a record player to the reflector of the transmitting klystron, the modulation being detected as music by aid of the speaker attached to the receiver. The combination of both kinds of modulation produced interesting effects, including the addition of artificial base notes to the music.

12. Hydrodynamic model of radioactive decay. HAROLD P. KNAUSS, *University of Connecticut.*—The well-known equations associated with radioactive accumulation and decay may be illustrated with containers of a special shape designed by Bohn and Nadig,[1] in which the volume of water represents the mass of a radioactive element remaining at each instant. In their design, the water flows out through a nozzle according to Torricelli's theorem; consequently the vessel has to have a volume proportional to the square root of the depth over the working range. If a capillary tube is used instead of a nozzle, the flow is proportional to the depth of water in the container according to Poiseuille's law, and an ordinary vertical-walled vessel may be used. A model was demonstrated.

[1] Bohn and Nadig, *Am. J. Physics* **6**, 320–3 (1938).

13. Laboratory experiment on alpha-particle scattering. ROBERT BERINGER, *Yale University.*—An alpha-particle scattering experiment patterned after that of Geiger and Marsden has been designed for an undergraduate laboratory course. A collimated beam of polonium alpha-particles is scattered in a vacuum chamber by a gold foil. Scattered particles are detected with a proportional counter-video amplifier-scaling circuit system. The experiment yields the angular dependence of the Rutherford scattering formula in the range up to 50° at counting rates of 1 min^{-1} or more. Background rates are much lower than this and permit a larger angular range for more extended counting periods. Students have encountered little difficulty with the experiment and show considerable enthusiasm. Professor E. Pollard is responsible for much of the apparatus design.

14. Nuclear magnetic resonance as an advanced laboratory experiment. ROALD K. WANGSNESS, *University of Maryland.*—Fundamental nuclear constants can be directly measured with relatively uncomplicated equipment in an advanced laboratory experiment using nuclear magnetic resonance. Several circuits have been described which can be adapted for this purpose.[1] The absolute value of, say, the proton magnetic moment can be found from its resonance frequency and the value of the constant magnetic field; the latter can be measured by standard flip coil methods. Ratios of other magnetic moments to the proton moment can be obtained directly from the ratios of resonance frequencies; a convenient nucleus for this purpose is F^{19}. The effects of external field inhomogeneities and paramagnetic ions upon line widths and relaxation times can also be demonstrated.

[1] N. J. Hopkins, *Rev. Sci. Inst.* **20**, 401 (1949); H. A. Thomas and R. D. Huntoon, *Rev. Sci. Inst.* **20**, 516 (1949).

15. On the use of curve differentials in thermodynamics. F. H. CRAWFORD, *Williams College.*—When the First Law

is written $dU = dQ + dW$, the notation on the right implies the existence of "heat" and "work" functions and this is contrary to fact. The use of a special symbol for d still leaves the meaning of the operations on the right of the equation obscure. For clarity we adopt a specific curve c and define both heat and work in terms of suitable integrals along c between a fixed and a *variable* point; since such integrals are functions of their variable (upper) limits, and these can be defined by a *single* variable, each integral is a function of a single variable. Hence we have a perfectly definite *heat function* and *work function* for the specific curve c. On differentiation of these functions we have the "curve" differentials dQ_c and dW_c. Thus, for example, if our curve be an isochore and we choose T as the independent variable for our integral, the first law would become $dU(T, V) = dQ_V(T) + dW_V(T)$, leading directly to a precise statement of the meaning of the specific heat at constant volume, for example, and the latent heat of expansion.

16. Transient analysis by a generalized symbolic method. Lucio M. Vallese, *Duquesne University.*—The transient response of linear electric networks with concentrated parameters is generally expressed with functions of time of types $Ae^{-\alpha t}\sin\omega t$, $Ae^{-\alpha t}\sinh\beta t$, and exceptionally $Ate^{-\alpha t}$. The analysis of the first type may be made conveniently with the use of the symbolic a.c. method; F. Creedy has shown[1] that the analysis of the second type of transients may be made with a similar symbolic method based on the vectorial representation of hyperbolic quantities. It was the purpose of this paper to show the application of these methods to the study of the transient response of a two-coupled loops network and of the density distribution of transient currents in the cross section of conductors. In the first case the response consists of two current components in each loop; the conditions which determine their dependence of time were given. In the second case it was shown that the distribution is quite different for circular- or for hyperbolic-exponential transients; in particular, no skin effect is present by an hyperbolic-exponential current.

[1] F. Creedy, *J. Math. Physics* 14, 291–324 (1935).

17. The graviton theory. William G. Graves, *Worcester Junior College*, with problems by Harold B. Burr, *Worcester Junior College.*—The graviton theory introduces a new idea of gravity and energy through the equation $L = [(2\pi)^2 a^2 + P^2] = $ constant, by unifying wave and corpuscular ideas. The word *graviton*, analogous to *photon*, represents the smallest independently moving mass with a natural, initial velocity greater than that of the accepted velocity of light. Energy is shown as paths cut by gravitons. Energy levels are determined by the variables in the three-dimensional L equation. Gravity gravitons at the lower energy level may then be shown to progress according to the L equation when the amplitude a approaches or equals zero. Both cause and effect of gravity and energy are integrated by the graviton idea; and through its application, problems are presented and solved in the related fields both of chemistry and physics.

18. A laboratory course in electronics. H. L. Schultz and W. G. Wadey, *Yale University.*—The nature of an electronics laboratory course given in the Physics Department for juniors and seniors was discussed. Emphasis is placed on the fundamental theory of operation of vacuum tubes and their uses as circuit elements in research applications. The equipment, which is largely home-constructed, is provided in a form which allows quick assembly by the student, and individual components are designed to provide maximum flexibility in setups. Several typical experiments were discussed in detail and illustrated by slides.

Attendance

The following persons registered at the annual meeting:

Vahan Ananikian, *University of Connecticut;* D. S. Ainslie, *University of Toronto;* Mildred Allen, *Mount Holyoke College;* Roland Allen; C. L. Andrews, *New York State College for Teachers;* Gladys Anslow, *Smith College;* Reuben E. Alley, Jr., *University of Richmond;* W. M. Baker, *University of Detroit;* Collis M. Bardin, *Compton College;* Le Roy L. Barnes, *Cornell University;* Henry A. Barton, *American Institute of Physics;* Paul F. Bartunek, *Lehigh University;* Louise G. Belai, *Our Lady of Cincinnati College;* Clarence E. Bennett, *University of Maine;* Earle M. Bigsbee, *University of Bridgeport;* H. Louisa Billings, *Smith College;* Jonathan Biscoe, *University of Maine;* Lester I. Bockstahler, *Northwestern University;* Robert A. Boyer, *Muhlenberg College;* Richard L. Brown, *Allegheny College;* Thomas B. Brown, *George Washington University;* Alfred K. Buchanan, *New Haven State Teachers' College;* George H. Burnham, *Norwich University;* W. G. Cady, *Wesleyan University;* Addison D. Campbell, *Hampden-Sydney College;* Isaac Chase, Jr., *Lowell Textile Institute;* W. L. Cheney, *George Washington University;* Edward P. Clancy, *Mount Holyoke College;* R. S. Cohen, *Yale* and *Wesleyan Universities;* Michael Coldwell, *Wesleyan University;* Donald B. Connelly, *Siena College;* Alfred Coutu, *Wesleyan University;* F. H. Crawford, *Williams College.*

F. E. Dart, *University of Oregon;* Frank T. Dietc, *Pennsylvania State College;* W. Lee Dorn, *Clarkson College of Technology;* Doan Douthett, *Vassar College;* H. E. Duckworth, *Wesleyan University;* V. E. Eaton, *Wesleyan University;* Stanley L. Ehrlich, *Underwater Sound Laboratory;* Fred K. Elder, *U. S. Navy;* Joseph D. Elder, *Harvard University;* Walter Eppenstein, *Rensselaer Polytechnic Institute;* H. A. Fairbank, *Yale University;* W. M. Fairbank, *Amherst College;* George Fisher, *Wesleyan University;* Howard M. Fry, *Franklin and Marshall College;* Walter L. Gerke, *Plainville High School;* A. T. Goble, *Union College;* Gary D. Gordon, *Wesleyan University;* S. O. Grimm, *Lebanon Valley College;* Albert N. Guthrie, *Brooklyn College;* V. B. Hall, *Wilkes College;* W. H. Hartwell, *University of New Hampshire;* Monica Healea, *Vassar College;* L. Grant Hector, *Sonotone Corporation;* Conrad Hemond, Jr., *Amherst College;* J. Barton Hoag, *U. S. Coast Guard Academy;* John R. Hobbie, *American International College;* James M. Holcomb, *St. Michael's College;* Harley Howe, *Cornell University;* Richard H. Howe, *Denison University;* Karel Hujer, *University of Chattanooga;* Hans Jaffe, *Brush Development Company;* Howard A. Johnson, *Wesleyan University;* Arthur Pierce Jones, *New York State College for Teachers;* Cort Kegley, *Wesleyan University;* Harold P. Knauss, *University of Connecticut;* W. D. Knight, *Trinity College;* A. A. Knowlton, *Bennington College;* Elizabeth R. Laird, *University of Western Ontario;* Joseph M. Lapetina, *Union University;* Edward M. Little, *University of Maryland;* Noel C. Little, *Bowdoin College;* John A. Lockwood, *University of New Hampshire;* Egon E. Loebner, *University of Buffalo;* R. A. Loring, *University of Louisville;* Kenneth V. Manning, *The Pennsylvania State College;* Alexander Marcus, *City College of New York;* Donald C. Martin, *Marshall College;* W. H. Michener, *Allegheny College;* Nora M. Mohler, *Smith College;* Burton Muller, *University of Illinois;* A. D. MacDonald, *Dalhousie University;* Robert MacDonnell, *Fairfield University;* Robert McCune, *Trinity College;* James H. McCloy, *Otterbein College;* Louise S. McDowell, *Wellesley College;* L. W. McKeehan, *Yale University;* Austin J. O'Leary, *City College of New York;* Herbert N. Otis, *Hunter College;* G. E. Owen, *Antioch College and Harvard University;* R. Ronald Palmer, *Beloit College;* R. F. Paton, *University of Illinois;* J. C. Pelton,

Jr.; R. J. Perry, *U. S. Coast Guard Academy;* Thomas D. Phillips, *Marietta College;* E. R. Pinkston, *U. S. Naval Academy;* F. F. Piper, *Clarkson College;* Eugene A. Plassmann, *Indiana University;* R. S. Preston, *Wesleyan University;* Robert Rea, *U. S. Coast Guard;* Claude Reed, *Union College;* R. A. Rhodes, *University of Connecticut;* Francis J. Rio, *Connecticut State Teachers' College;* Eric M. Rogers, *Princeton University;* Duane H. D. Roller, *Harvard University;* Duane Roller, *Wabash College;* Yale Kirby Roots, *New York University;* William H. Ross, *University of Massachusetts;* H. L. Schultz, *Yale University;* Francis W. Sears, *Massachusetts Institute of Technology;* Sister Ann Cecilia, S.S.J., *College of Our Lady of the Elms;* Sister Teresa Marie, S.S.J., *College of Our Lady of the Elms;* Robert S. Shaw, *City College of New York;* Harold E. Smith, *University of Bridgeport;* Theodore Soller, *Amherst College;* A. H. Spees, *University of Connecticut;* George Stanford, *Wesleyan University;* M. N. States, *Central Scientific Company;* Elva Stearns; W. W. Stifler, *Amherst College;* Harry Sussman, *Underwater Sound Laboratory;* Richard M. Sutton, *Haverford College;* Thomas

E. Thomas, *Wilkes College;* Frank H. Todd, *University of Maine;* Emory C. Unnewehr, *Baldwin-Wallace College;* K. S. Van Dyke, *Wesleyan University;* Van Der Sluis, *The Pennsylvania State College;* Mario P. Vano, *University of Detroit;* W. G. Wadey, *Yale University;* Hugh F. Walsh, *Siena College;* Bernard B. Watson, *U. S. Office of Education;* Roald K. Wangsness, *University of Maryland;* H. E. Way, *Union College;* R. C. Weaver, *Virginia Military Institute;* M. Russell Wehr, *Drexel Institute of Technology;* Gertrude B. Wertenbaker, *Hunter College;* George F. Wheeler, *Physics Films Company;* James W. White, *University of Tennessee;* Mrs. J. Harvey White; Robert A. Willett, *Wesleyan University;* Ellis D. Williams, *York Junior College;* Charles Williamson, *Carnegie Institute of Technology;* Hugh C. Wolfe, *Cooper Union School of Engineering;* Richard F. Woodcock, *Wesleyan University;* Karl S. Woodcock, *Bates College;* Ralph S. Woollett, *U. S. Navy;* D. C. Worth, *Nanking University;* Mark W. Zemansky, *City College of New York.*

RECENT MEETINGS

Colorado-Wyoming Section

The annual meeting of the Colorado-Wyoming Section was held at Colorado A. and M. College, Fort Collins, Colorado, in conjunction with the twenty-first annual meeting of the Colorado-Wyoming Academy of Science, held May 12 and 13, 1950. The following program was presented:

Bell system of television network facilities (invited paper). M. E. Strieby, *Bell Telephone Company.*

Mathematical physics for the potential graduate student. John Phelps, *Colorado College.*

A technique for taking data on the critical potentials of gases. Robert S. Bradford, *University of Wyoming.*

A vacuum tube electrometer for student use. Robert O. Bock, *Colorado A. and M. College.*

Experiments with one-fortieth farad capacitor units. Frank P. Goeder and Louis R. Weber, *Colorado A. and M. College.*

Teaching of the limit of resolution of telescopes. S. W. Harding, *University of Wyoming.*

Reflectance of polarized light by direct method for the optics laboratory. Peter Jowise and John Phelps, *Colorado College.*

The newly elected regional chairman for 1950–51 is Dr. Robert O. Bock, Department of Physics, *Colorado A. and M. College,* Fort Collins, Colorado. All correspondence concerning the Section for the ensuing year should be directed to him.

C. A. Cinnamon, *Chairman*

Oregon Section

The Oregon Section of the American Association of Physics Teachers held its fifty-fourth session on May 13, 1950 at the University of Portland, Portland, Oregon. Greetings of the University of Portland were expressed at a luncheon by the Rev. Robert H. Sweeney, C.S.C., Vice-President of the University. Arrangements for the

meeting were made by Brother Godfrey Vassallo, *University of Portland.* A tour of the new Engineering Building was conducted by Brother Godfrey and a special laboratory demonstration, "A Method of Measuring the Velocity of Sound" by Mr. Don Graham, was provided.

The business session was called to order at 3:45 P.M. by Dr. W. R. Varner, *Oregon State College,* President of the Oregon Section. Dr. W. V. Norris, Representative to the National Executive Committee, reported on several matters: American Institute of Physics abstracting of papers; the 12,500 membership in the American Institute of Physics, representing a 100 percent increase in the last ten years; the meetings of the Association scheduled in January at New York and in October at Chicago; the status of *Physics Today* as official organ for the American Institute of Physics; and the financial condition of the Association. Dr. C. L. Utterback announced seminars to be held at the University of Washington beginning July 5, 1950, and conducted by Dr. Hans Bethe. Dr. M. A. Starr announced the installation of a Sigma Xi Club in Portland.

Officers for the year 1950–51 were elected as follows: *President,* Dr. Merle A. Starr, *University of Portland; Secretary,* Mr. Fred W. Decker, *Oregon State College; Historian,* Brother Godfrey Vassallo, *University of Portland; National Representative,* Dr. W. V. Norris, *University of Oregon.*

Invitations were accepted to hold meetings in 1950–51 at the State College of Washington in the fall, at Linfield College in the winter, and at Reed College in the spring. Dates are to be proposed by the Executive Committee.

The following members registered for the fifty-fourth meeting of the Oregon Section of the American Association of Physics Teachers:

College of Puget Sound: George Bjorke, Vernon E. Calloway, Ralph C. Eikenberry, Allen R. Mantheir, Richard B. Nicholson, Hsi-lung Pan, Raymond S. Seward; *Iron Fireman Manufacturing Company:* R. H. Webster; *Lewis and Clark College:* A. A. Groening, J. H. Karle, D. D. Loomis, Eldred Toole; *Linfield College:* George

Barnes, Walter Dyke; *Oregon State College:* Duis Bolinger, Cleo C. Byers, C. L. Church, M. V. Davis, John Day, Fred W. Decker, John Garman, H. R. Kaiser, R. L. Lincoln, R. P. Merritt, W. R. Varner, C. T. Wang, W. Weniger, E. A. Yunker, C. J. Filz, J. W. Griffith; *Reed College:* K. E. Davis, L. S. Germain, W. L. Parker; *Tektronix, Incorporated:* Howard Vollum; *University of Oregon:* A. E. Caswell, F. E. Dart, S. Y. Chin, R. T. Ellicksen, W. V. Norris; *University of Portland:* G. Godfrey, W. J. Keane, R. D. Murphy, M. A. Starr, Paul E. Wack; *University of Washington:* J. J. Faris, J. E. Henderson, Harold N. Ritland, C. L. Utterback; *Vanport Extension Center:* Allan Gibb, T. A. Shotwell, R. B. Walton; *Washington State College:* R. M. Brown, Alfred B. Butler, K. E. Fitzsimmons, A. R. Tobey, S. T. Stephenson; *Willamette University:* E. T. Brown, J. F. Carpenter, Laurence Cherry, K. G. Clemann, A. C. Gardy, F. N. Gruvur, C. T. Luther, R. L. Purbrick.

A program of invited and contributed papers was presented as follows:

The lost Port Orford meteorite. J. HUGH PRUETT, *American Meteor Society* (read by FRED W. DECKER).—In 1859, Dr. John Evans, a government geologist, found an unusual rock protruding from the ground "on a bald mountainside about 40 miles from Port Orford, Oregon." Specimens analyzed in New York and Vienna were pronounced meteoritic. Plans to move the mass, estimated at "fully 10,000 kilograms," to the Smithsonian Institution at Washington came to an end with Dr. Evans' death in 1860. The lost Port Orford meteorite has ever since been an object of intensive search. The only known specimens are in the Imperial Mineral Cabinet in Vienna and the Smithsonian Institution.

Meteorological aspects of the fluorine problem. JOHN A. DAY, *Oregon State College.*—This fluorine problem arose as a result of the establishment of three large aluminum plants in the lower Columbia River valley. A physioclimatological approach was followed. The plant physiologist was requested to specify limits of (a) temperature, (b) humidity, and (c) radiant energy within which specific plants would show maximum absorption of F. Hourly weather data prepared on IBM punched cards was used. These cards were sorted mechanically, discarding all which did not fall within *all* the limits specified. The remaining cards were sorted by wind octant. The octant lying ESE-SSE of a source of F was seen to be most favorable. The method of approach seems to have possibilities for use in other studies.

Some problems in crystal growing. HENRY KAISER, *Oregon State College.*

The mass of the meson. L. S. GERMAIN, *Reed College.*—Nearly all measurements of the mass of the meson involve a measurement of the momentum of the particle and a measurement of some quantity such as range or specific ionization which is an independent function of the velocity. It has only been within the last five years that experiments to find meson mass measurements in quantity have been carried out. The first such experiment was performed by Fretter, who with the aid of two cloud chambers was able to make simultaneous measurements of momentum and range and found mass values for 26 cosmic-ray mesons. These values average 212 electron mass units. The experiment has been improved and repeated by others with substantially the same results. The production of mesons by high energy alpha-particles and protons has opened up another method of measuring meson masses in quantity. Measurements of momentum and range in photographic emulsion have been made for a number of π-, μ-, and n mesons. These measurements have given $m = 276$ and $m_n = 210$. Experiments are in progress at the Radiation Laboratory in Berkeley to compare m and m_n to the mass of the proton with great accuracy.

Small angle x-ray scattering. HAROLD RITLAND, *University of Washington.*—The study of the scattering of x-rays at very small angles offers information on the size and shape of the submicroscopic scatter particles. The theory developed by Guinier and others was briefly reviewed, and an application of the method to the study of protein molecules in solution was described. Radii of gyration and axial ratios were obtained for five proteins. The relation of these results to the problem of hydration in protein solutions was discussed.

Laboratory techniques for seniors in physics. K. E. FITZSIMMONS, *State College of Washington.*—A laboratory course, to give college seniors in physics an acquaintance with some of the techniques encountered in research, has been offered at Washington State College during the current year. Emphasis has been made on the techniques although, once an experiment has been selected, precision measurements are encouraged wherever possible and necessary. The choice of part of the experiments was so made that the regular senior laboratory course offered in electron physics could be eliminated. The students have been encouraged to use their imagination and were required to aid in setting up the experiments performed. Considerable interest has been shown by the present senior group. The course not only exposed them to research techniques generally for the first time, but served to indicate their aptitudes for experimental work.

The first electrical experiment in sophomore physics. ALFRED B. BUTLER, *State College of Washington.*—An experiment is described in which the primary object is to have the students become acquainted with electrical apparatus and its symbols, and to gain facility in setting up electrical circuits. Ten different circuits are connected by the students using dummy batteries. Each circuit connection is checked by the laboratory instructor. Some circuits are used which they will have in later experiments. Other circuits must first be completed on paper by the student and then wired with the electrical components. Student and staff reaction both has proved to be highly favorable to this experimental period.

The chronology of modern physics. A. R. TOBEY, *State College of Washington.*—Our physics majors are going to be called on more and more to interpret physics to the world at large as a force in the contemporary social, political, and economic scene. They must answer "Where is physics going?", and hence, must know "Where is physics now?", and "Where has physics been?". Traditional treatments fail to provide him with the bases for the answers to these questions. The separate "History of Physics" courses fail to produce the necessary integration. Modern Physics, drawing on all areas of classical development, provides an excellent opportunity to present a rigorous treatment in a modified historical format. The advantages of such a treatment are manifold: The ideas of physics are presented with their origins. The nature of constructs resulting from indirect reasoning is clearly emphasized. The dynamic nature of physics is inescapable. Orientation of the various areas of physics in relation to contemporary problems is accomplished. The chronological order of business usually turns out to be as good a logical order as any alternate arrangement. Vague references to future material to be covered necessary to an immediate argument are avoided. Indelible impressions of inversions in history are not created in the student's mind. The drama of human endeavor in the development of physics increases student interest.

Amplifiers for cathode-ray oscilloscopes. HOWARD VOLLUM, *Tektronix, Incorporated.*

An inexpensive three-meter diffraction grating spectrograph. GEORGE BJORKE, *College of Puget Sound.*—The spectrograph was constructed with a replica grating of three-meter focal length. A modified Paschen mounting with normal incidence was utilized. The outstanding features of the spectrograph are: (1) A camera which may be moved any place in the focal plane by means of an electric motor. The camera is equipped with a shutter and a self-removal device for the dark slide on the plate holder. (2) A light box which contains the light source and condensing lens and slit. It also is equipped with a shutter. The spectrograph also features electric focusing of the grating.

Physics then and now. A. E. CASWELL, *University of Oregon.*—The paper dealt with the remarkable advances in the field of physics between the discovery of x-rays in 1895 and the present time. Beginning with a few personal reminiscences, the speaker read a number of quotations from the *Scientific American* of 1849 and 1899 illustrating the remarkable changes that have taken place in many other fields during the nineteenth century. The historical development of physics over the centuries was rapidly sketched and the modern developments were considered. The speaker pointed out that the discovery of x-rays marked the beginning of an era in physics just as Columbus's discovery marked the beginning of an era of exploration. The greater part of the physics taught in the graduate schools today was unknown to the professors of physics in 1895. This is the golden age of physics and we do not

know what discoveries await us, but we look forward to the future with anticipation. The demand for trained physicists is greater than ever before and may be expected to continue so for some time, but on the other hand, the graduate departments of physics are following the lead of the medical schools in raising the requirements for admission to the profession.

Field emission at large current densities. WALTER P. DYKE AND J. KENNETH TROLAN, *Linfield College* (presented by DR. DYKE).

FRED W. DECKER, *Secretary*

Western Pennsylvania Section

The spring meeting of the Western Pennsylvania Section of the American Association of Physics Teachers was held May 13, 1950 at Waynesburg College, Waynesburg, Pennsylvania. Thirty members and guests attended the meeting. A. J. KAZORA, President of the Section, presided. After greetings by the president of Waynesburg College the following papers were presented:

The need for an improved program of training high school physics teachers. GORDON M. DUNNING, *State Teachers' College, Indiana, Pennsylvania.*—Specific recommendations concerning state certification requirements for high school physics teachers could be made to state authorities if a study of what constitutes good high school physics teacher preparation were inaugurated by the Section. College teachers need to recognize that not all physics courses are equally valuable for the future research physicist and the prospective teacher. There is a need for graduate courses which lie within the comprehension of and which are of specific value to teachers. College physics teachers and students need to become better acquainted with high schools and their work. Students should be prepared to teach physics relevant to living in 1950.

Some musings of an ex-lab instructor. F. A. MOLBY, *professor emeritus, West Virginia University.*—As a result of regimentation of workers during the war and of the regimentation which may attend federal support to research in colleges and universities there are dangers in independent work in research in physics. It is feared that assembly line procedures, such as wartime methods for mass-training in physics and for use with the postwar influx of students, may persist after the need for them has passed. Many top American physicists have originated in the freedom of small colleges.

A phototube-controlled slave flashgun. H. F. OSTERMAN, R. W. ASHBEE AND C. WILLIAMSON, *Carnegie Institute of Technology.*—Photographers often require more than one flashbulb to illuminate an interior view. The extra flashguns are usually synchronized by wire connections. Recently designs for phototube flashguns have been published. Most of these use mechanical relays. Our circuit uses a 2D21 thyratron to fire a No. 25 Sylvania flashbulb. Energy for firing is stored in a one-microfarad condenser charged

to 135 volts. Four flashlight batteries supply filament power for the miniature thyratron. A tiny 22½-volt battery biases the thyratron control grid. The phototube is a Type 930. The entire circuit goes into a cubical box five inches on an edge, with room to spare. The total cost of parts is under $20. A neon lamp in parallel with the flash-bulb socket enables one to test the circuit without wasting a flashbulb. A pilot lamp warns the operator if the filament power is inadvertently left on.

Differentiated physics courses at the University of Pittsburgh. Oswald Blackwood, *University of Pittsburgh.* —General physics is offered at three levels: engineering physics with calculus corequisite; general physics, for pre-medical students, and the like; nontechnical physics for students who think that it will have no professional value. Two second-year courses in atomic physics are offered, one for physics majors, the other for the general student.

Mirage, or regular reflection? Cecil O. Riggs, *Waynesburg College.*—The motorist's experience of seeing water on the highway together with reflections of cars and roadside objects is probably due to regular reflection by the road surface rather than to a mirage. Many diffuse reflectors become excellent regular reflectors at grazing incidence. The regular reflection explanation is supported by observations that the images are always inverted, are not shimmering, can occur on windy days, and are not affected by traffic. A simple calculation assuming a fictitious refracting prism requires an impossible differential in indices, but a more rigorous mathematical examination will probably not preclude mirage formation.

Physics laboratory arts; an undergraduate course. Harry Hill, *Washington and Jefferson College.*—A laboratory course for physics majors was outlined which consists of general laboratory techniques the first semester. The individual assignments, based on a student's answers to a long list of questions, may be instrument repair, shop practice, Pyrex and plate glass working, high temperature measurements, photography, photoreflex printing, making and calibrating thermocouples, cleaning mercury, high vacuum, silvering mirrors, use of special apparatus, etc. There is a period of discussion before each laboratory period. Discussion periods continue during the second semester but each student works on some project of a pre-research nature, working out his own methods and finding out where and how to secure materials.

Preparing rods for stroking (Kundt's tube). Bernard L. Brinker, O.S.B., *St. Vincent College.*—An end section of a rod can be quickly coated with a hard film of rosin by a simple dipping-and-curing procedure. Such a film has excellent properties for vibrational stroking in Kundt-tube studies or demonstrations.

The business meeting followed luncheon on the campus and a visit to the physics laboratories. Reports of Fr. Bernard Brinker, representative to the Executive Committee of the Association, on current activities of the AAPT and of O. Blackwood on the annual meeting of the Executive Committee were received. In accordance with the desire of the national secretary, it was decided that each member presenting a paper should submit an abstract of not over 100 words to be included in the records of the Section and to be submitted to the *American Journal of Physics* for possible publication. A committee consisting of Gordon M. Dunning, *State Teachers College, Indiana, Pennsylvania,* O. Blackwood, *University of Pittsburgh* and James M. Smith, *Thiel College* was appointed to study proposed certification changes for high school physics teachers in Pennsylvania and to prepare resolutions for adoption by the section at its fall meeting, such adopted resolutions to be presented to state authorities as recommendations of the section.

The fall meeting of the section will be held at Carnegie Institute of Technology.

R. E. Warren, *Secretary*

American Society for Engineering Education Physics Section

The summer meeting of the Physics Section of the American Society for Engineering Education was held jointly with the American Association of Physics Teachers on June 19–23, 1950, at the University of Washington, Seattle, Washington. Chairmen of the various sections dealing with physics were: Professor G. P. Brewington, *Lawrence Institute of Technology,* Professor C. L. Utterback, *University of Washington* and Professor C. R. Wylie, Jr., *University of Utah.*

The following papers were presented:

Accelerators of nuclear particles. John S. Streib, *University of Washington.*

Ramjets and turbojets. J. E. Dwinnell, *University of Washington.*

Some uses of the electron microscope in engineering. Oliver Row, *University of Washington.*

An analysis of laboratory instruction in physics. Smith E. Colwell, *South Dakota School of Mines & Technology.*

Employment of physicists and mathematicians in industry. G P. Brewington, *Lawrence Institute of Technology.*

Computing machines and their applications. H. D. Huskey, *National Bureau of Standards Institute for Numerical Analysis.*

The mathematics of plastic flow. Peter Gibbs, *University of Utah.*

The teaching of mathematics of airflow theory. Robert L. Street, *University of Washington.*

Officers elected by the Physics Division of the American Society for Engineering Education were: *Chairman,* Elmer Hutchisson, *Case Institute of Technology; Vice-Chairman,* F. G. Slack, *Vanderbilt University; Secretary,* G. P. Brewington, *Lawrence Institute of Technology; Representative on General Council,* C. E. Bennett, *University of Maine; Members of Executive Board,* J. G. Potter, *Agricultural and Mechanical College of Texas,* H. L. Dodge, *Norwich University,* and Marsh W. White, *The Pennsylvania State College.*

G. P. Brewington, *Secretary*

ANNOUNCEMENTS AND NEWS

Book Reviews

Introduction to Theoretical Physics. MAX PLANCK. Vol. I, General Mechanics. Pp. 272, Figs. 43. Price $3.00. Vol. II, Mechanics of Deformable Bodies. Pp. 234, Figs. 12. Price $3.00. Vol. III, Theory of Electricity and Magnetism. Pp. 247, Figs. 12. Price $3.00. Vol. IV, Theory of Light. Pp. 216, Figs. 24. Price $2.75. Vol. V, Theory of Heat. Pp. 301, Figs. 7. Price $3.25. $5\frac{1}{2} \times 8$ in. The Macmillan Company, New York, 1949.

As the titles suggest, these five volumes are devoted to what has come to be the standard subject matter of the "theoretical physics course." Indeed, these volumes in their earlier editions—the first edition was in 1916, the latest (of which this edition is a reissue) was 1928—may well have delineated the repertoire for the many subsequent books on theoretical physics. The author, Max Planck, was not only the founder of modern quantum theory, but also (as he recounts in his *Scientific Autobiography*) the first purely theoretical physicist of his time. Since he also contributed considerably to the development of classical theoretical physics, it is of particular interest to examine these books in the light of the subsequent developments which he helped initiate in both the science and pedagogy of physics.

The first impression of the experienced reader on perusing the contents of these books is likely to be one of disappointment. On the formal side, the treatment seems elementary; there is no evidence of powerful mathematical techniques or elegant artifices. The less mathematically minded physicist can legitimately object that the exposition makes little contact with atomic or nuclear physics. As possible texts or reference volumes, these books have the serious shortcoming of being neither comprehensive nor up-to-date. A more serious criticism of this work as a text comes from the fact that nowadays one has come to expect of the theoretical physics course that it should both prepare the student to deal with problems in classical physics and at the same time develop in him a familiarity with methods and ways of looking at things which will prove fruitful in modern physics. Unfortunately, these volumes of Planck do not attempt to face this latter requirement; in fact their early vintage, 1916–1928, ironically precludes any accommodation to the many developments in atomic, molecular, and nuclear physics which now dominate contemporary physics. Thus there is no treatment at all in Vol. I, General Mechanics, of coupled oscillators or normal modes, topics which are indispensable for even the simplest molecular models. The brief treatment of collisions does not at all prepare the reader for understanding atomic or nuclear scattering experiments—Rutherford scattering is nowhere mentioned. The chapters on optics were written too early to stress the analogies in wave optics and wave mechanics which can make plausible the uncertainty principle. Most of the electricity and magnetism volume is devoted to stationary or quasi-stationary fields and macroscopic bodies; applications to radiation, atomic electro-

magnetic interactions or boundary value problems are conspicuously absent. On the other hand, the treatments of deformable bodies and the theory of heat, while not pretending to be comprehensive, are masterpieces of elementary exposition of traditionally difficult subject matter. Indeed, the reviewer is not aware of any disucssion (in English) of the classical theory of blackbody radiation which is presented with comparable insight and clarity.

These remarks might tempt one to conclude that this work of Planck is outmoded today. On closer examination such a conclusion appears both unjust and irrelevant; it misses the whole point of these books. In the first place, Planck deliberately limited the choice of material almost wholly to the classical theory of the physics of the continuum. And even within this restricted framework he admits to having dealt with "only a meagre selection from the very abundant material available." He was not trying to write a definitive treatise nor a textbook of what every student of theoretical physics should know then or now. Rather his aim was to help the student to think physically and he regarded this work as a contribution to the teaching of physics. Thus, in the preface to Vol. I he says:

> During my long activity as a teacher I have frequently observed that the difficulties with which the student has to contend when he first enters the realm of theoretical physics are more often not concerned with the mathematical form, but with the physical ideas which are presented to him. It is not the calculations with equations that cause him most trouble, but the setting up of the equations and, in particular, their interpretation. The chief purpose of the present volume is to lend him a helping hand in this respect···. The student is not···led along in the direction traditionally prescribed by the classical writings of science, but rather is advised and occasionally warned at the decisive turning points, in order that something of that particular pleasure may be retained which every person of independent thought experiences when advancing for the first time into a new field of science.

This concern for the student's understanding is apparent throughout these volumes. The student will find each topic treated clearly and leisurely, the explanations often anticipating his questions, and more often raising questions which more glib treatments might ignore. Mathematics and formal manipulations are kept in the background; indeed, the notation is not always as streamlined as one might wish, but this also has its advantages in that it forces the student to establish for himself just how much of the argument is physics and how much mathematics. The printed pages read more like discursive essays: each point is probed thoroughly to bring out the essential physical content. The reader is often impressed with the distinction between the mathematical description and the actual physical situation (for example, "we must bear in mind that in optics we never observe a single system of

parallel plane waves···but always a narrow cone of ray directions").

In respect to meeting the student at an accessible level, motivating his interest in the problems of classical physics and guiding him to deeper physical understanding, these volumes of Planck have yet to be excelled. They are moreover written in a style which reflects a certain simplicity and humanity in the author which commands the reader's confidence and interest. This is in pleasant contrast to the dry style of most physics literature, whatever the intended level. Granted that not every author can aspire to the human or scientific stature of Planck, it seems more probable that the main reason for the traditional soporific style of physics writing is the unfortunate misconception that it is either in bad taste or incompatible with scientific expression for a scientist to display enthusiasm, literary style, or, in general, any thing but facts in their barest form in his scientific writings. Fortunately, Planck transcended this narrow view and as a result these unpretentious books can serve as a model for physics books whatever their subject matter. Indeed, they do admirably succeed in fulfilling Planck's professed purpose in writing them: to provide a helpful and stimulating companion for any physicist's introduction to theoretical physics.

DAVID L. FALKOFF
University of Notre Dame

Joseph Henry, His Life and Work. THOMAS COULSON. Pp. 352. Princeton University Press, Princeton, New Jersey, 1950. Price $5.00.

In all of the history of physics, there are few individuals whose contributions are more difficult of evaluation than are those of Joseph Henry. His contemporaries in Europe extended him little recognition; even in his own country there was greater appreciation of the "practical" Samuel Morse than there was of the teacher whose work contributed so greatly to electrical communication; historians have tended to minimize Henry's talent in comparison with the greater genius of Faraday. Yet one cannot read Henry's scientific writings without realizing that his ingenuity in devising experiments and his clear thinking in the analysis of the results made him a figure of the first magnitude in early nineteenth century physics.

Coulson's study of Joseph Henry is too enthusiastic and partisan to be a definitive work, but it does help the reader to an understanding of the conditions and the difficulties under which Henry labored, and serves to place him among his contemporaries, particularly those in America. In summarizing his own conclusion the author says:

Henry lacked the divine spark of the superlative genius. He too rarely abandoned patient research in order to follow a brilliant intuition. Like a child he went on asking "Why?" without seeking a final answer. With the naivete of childhood he kept his freshness of outlook, and this seems to have encouraged him to plunge into new situations with zest. As a result, his scientific life was a series of beginnings.

A comparison of Henry's papers on electromagnetism and electromagnetic induction with those of Faraday convinces one that Coulson has here found a key to the puzzle.

In his 1832 paper, which antedates any work of Faraday on self-induction, and even in his 1835 article, in which he amplified the statement of his experimental results, Henry nowhere grasps the clear connection between self- and mutual-induction which Faraday displays in Series IX of his *Experimental Researches* (1834). These same publications, however, also demonstrate that the poor eastward diffusion of scientific communications played an appreciable role, since Henry quoted Faraday's 1834 work in February of 1835, while the British physicist gives no indication of having seen the American publication of 1832. His only acknowledgment of early observations is a brief reference to Jenkin.

One contribution of Coulson's book lies in the emphasis placed on those parts of Henry's researches which are not commonly associated with his name. Among these are his development of circuit matching, his recognition of electrical oscillations, his production of noninductive windings, and his partial recognition of a conservation law for "imponderables." The first of these discoveries was the more remarkable for the fact that it was independent of, although subsequent to, Ohm's famous paper. In connection with the discussion of imponderables, the reviewer cannot agree that a study of Henry's work "shows him to have been among the very first to form clear ideas of the correlation of forces which extended beyond a mere transformation of one force into another, as magnetism into electricity, or electricity into light." His 1851 paper does not show the keen analysis found in the contemporary writings of Mayer and Joule, and is indicative of the essentially qualitative nature of his work.

About half of the biography is devoted to Henry's more productive scientific years, spent in Albany and Princeton, while the rest of the book deals with the administrative phase of his life. The latter part will not be of as much interest to the student of physics as is the earlier portion, but the early history of the Smithsonian Institution and of the National Academy of Sciences is not without its lesson for a period when the overlap of science and government has become a major concern of physicists. One cannot read this section of the book without feeling that, regardless of the progress of physics in the last century, the relation between the physicist and the politician, or between the physicist and the soldier, has shown little change.

WALTER C. MICHELS
Bryn Mawr College

The Cyclotron. W B. MANN. Pp. 90, Figs. 31, $4\frac{1}{4} \times 6\frac{3}{4}$ in. Methuen and Company, Ltd., London, 1940. Price $1.25.

This little book in the Methuen series of monographs, first published in 1940 in Great Britain, and now being republished in this country, is written primarily for the "Honors Student in Physics," although it will prove useful to teachers of certain undergraduate physics courses and those in allied fields who may be interested in obtaining a better grasp of the principles and general methods of construction of cyclotrons. The book is written in a clear, readable, and interesting style. Dr. Mann introduces the

subject by presenting a brief account of the reasons for the development of high-voltage accelerators and the problems associated with them which led E. O. Lawrence to his ingenious idea of the cyclotron. One chapter is devoted to a discussion of the conditions requisite for magnetic resonance acceleration, while subsequent chapters discuss the principal units of the cyclotron: the vacuum chamber and magnet, the high-frequency supply and ion source, electrostatic and magnetic focusing, and necessary adjustments in the cyclotron. Finally, a very brief but indicative discussion is given of the application of the cyclotron to problems in physics, chemistry, biology, and medicine. The numerous references will enable the reader to pursue further, if he so wishes, the historical development and technical details.

The descriptive material is based largely on the 37-inch machine at the University of California, although numerous references are made to their 60-inch cyclotron. which was put in successful operation while the book was in press. Of course, at that time the great 184-inch synchro-cyclotron existed perhaps only as a hope in the mind of Lawrence.

The foreword by Professor Lawrence is in itself interesting, particularly in retrospect.

One would not be justified in making adverse criticism of this little book since it does accomplish just what it attempted. The technical details appear to be completely correct, and the descriptions are very clear. In addition, and equally important, it has captured a certain amount of the spirit of cyclotron research.

W. C. PARKINSON
Univrsity of Michigan

Physics. OSCAR M. STEWART AND NEWELL S. GINGRICH. Fifth edition. Pp. 726+viii, Figs. 451, 15½×23½ cm. Ginn and Company, New York, 1950. Price $5.00.

This textbook was first published in 1924 and has now reached its fifth edition. It has had an unusually long and successful life and there can be few instructors who have not had some contact with it. It is one of the easier textbooks of college physics. The mathematics needed for its study is limited to ordinary arithmetic and algebra as far as the manipulation of simple equations. Trigonometry is used for the resolution of vectors, but all that is needed is the definition of sine and cosine, which is given in the text, and a little practice in reading the values of these functions from a table. The logarithm is mentioned once only, in connection with the decibel, and its meaning is given in a footnote. A single year of high school algebra and geometry combined would appear to be a sufficient mathematical preparation.

This limitation means, inevitably, that explanations are verbal, and sometimes verbose; and that more difficult topics must either be omitted entirely, or referred to with the use of the phrase "it may be shown that" or some equivalent.

The problems are numerous. They are largely arithmetical in nature, and some are solved by mere substitution in a formula. However, they do serve to give a valuable drill in the use of the principles, and at this level, little more can be expected. The problems are nearly all

changed from the fourth edition. The changes are often mere numerical ones, and probably serve no other purpose than the commercial one of making the fourth edition unusable.

The first 38 chapters have excellent summaries. Chapter 39 (there are 44 altogether) tells the student to make his own summaries for the rest of the book, and says that this may be more valuable than the use of printed summaries. Perhaps it was a pity to deprive him of this valuable opportunity till such a late point in his course.

The book is eminently suitable for a freshman course for students without strong mathematical preparation, with little or no high school physics, and with small desire to pursue the subject further. For premedical students, and others whose desire is to satisfy a requirement as effectively and as painlessly as possible, it is ideal. It is not sufficiently complete nor detailed for use by pre-engineers or well-prepared physics majors. It is a straightforward and simple exposition of the principles of physics and does not pretend to have any specialty of history or philosophy of physics, or any of the other attitudes now prevalent in books of "science in general education"—though it does have an excellent appendix article on scientific method.

The development of the subject is entirely conventional. The easiest topics are put at the beginning, letting the student into his subject very gently, allowing him time to get used to all his new surroundings before facing him with any real difficulties. There is, perhaps, a slight disadvantage in that there are some breaks in continuity between the early topics, but these breaks have to come somewhere.

Throughout mechanics, three systems of units are carried along, both in text and in problems—the cgs, the mks, and the British Engineering system in which force, rather than mass, is regarded as fundamental. In the B.E. system, of course, the unit of force is the pound, defined for the U. S. as being 1/2.20462 of the weight of a kilogram: the unit of mass is a derived one, the slug. The authors themselves are aware that in the systems used the process one goes through with a beam balance should be described as a comparison of *masses* when either of the metric systems is used; but as a comparison of *weights* when the B.E. system is used; and that in calorimetry the pound is used as a unit of mass, being then 1/32 slug. These difficulties are clearly pointed out and are only examples of the consequences of a multiplicity of unit systems.

In electricity, the approach is still traditional, by way of magnetic poles to magnetic fields, static charges, electric fields, potential, condensers, to electric currents and circuits. In magnetism, the problems are set in the cgs absolute electromagnetic system. In electrostatics the cgs absolute electrostatic system is used, and for currents the practical or mks system.

Modern physics is neither neglected nor emphasized to the detriment of classical physics. There are two chapters on atoms, electrons, and nuclei, and numerous discussions of modern topics at places in the text where the principles involved find their natural place.

The fifth edition is a slightly smaller book than the

fourth—726 pages against 785. Nevertheless, there have been considerable additions, which definitely modernize and improve the book. Notable additions are descriptions of the Van de Graaff generator, the cyclotron, the betatron, the mass spectrograph, a five-page treatment of alternating currents, and a discussion of the Bohr atom. Many topics have been rewritten, including those on atomic physics. Trigonometry has been introduced three chapters earlier than before, to allow its use in the resolution of forces, a most desirable change. The chapter on elasticity has been moved earlier to precede that on vibrations, and the enlarged chapter on capacitance has been placed next to the one on electrostatics. To gain space for these changes, there have been some omissions and compressions. For example, the applications of heat to meteorology have been cut down. Little appears to have been lost by the omissions, whereas the additions seem vitally necessary, and the rearrangements highly desirable.

The changes have not altered the character of the book at all. Almost anything that could be said of the fourth edition remains true of the fifth. Its long life shows that it has been popular, and the changes will help to maintain this popularity.

REGINALD T. HARLING
St. Lawrence University

University Physics. F. W. SEARS AND M. W. ZEMANSKY. Pp. 848. Addison-Wesley Press, Inc., Cambridge, 1949. Price $6.00.

This text is in essence a condensation of the well-known three-volume series, *Principles of Physics* by F. W. Sears, but whereas the extended series was intended for a two-year course, the present volume is more realistic in recognizing that in many universities not more than one year is allocated to this basic study in the engineering and science programs. Presumably this realistic approach has also deterred the authors from attempting to make the book encyclopedic. The approach is essentially to emphasize a few fundamental principles and develop these as extensively as possible in a general course. Use is made of the calculus, presumably to the extent that the student becomes acquainted with this branch of mathematics in a concurrent course.

The section on mechanics opens with a statement of Newton's first and third laws using the topics of center of gravity and conditions of equilibrium as illustrative material. The customary preference to begin mechanics with the study of dynamics is based partly on logical grounds and partly on student interest. The use of statics in this text as a jumping off point is undoubtedly motivated by a desire to mark time while the student is being initiated into the use of calculus. Kinematics follows and the second law is introduced with an exposition of both the cgs and mks units of force. The attendant dilemma of the fps system is resolved in favor of the slug. It is commendable that the principles of energy and momentum are drawn in at an early stage and advantageously utilized thereafter.

The requirements of clarity in exposition and meticulous care in development of ideas are well met by this text.

The style tends to the informal, and often there are explicit warnings against the type of misjudgments commonly encountered among the students. The book is replete with good diagrams and photographs. Particular mention may be made of the set of multiflash photographs which help to visualize the significance of the center of percussion and other dynamic cases.

The sequence of deductive reasoning used in the section on mechanics is well maintained in other parts of the book. The units employed in the section on electricity are in the main the rationalized mks system with a parallel discussion of the cgs system at many important stages. Coulomb's law, electric fields, and electric potential are extensively and logically developed and are followed by a discussion of dielectrics and polarization. Use is made of these concepts later in explaining the mechanism of dissociation and conduction in electrolytes. The magnetic field is introduced through the force on a moving charge and continuity is maintained by using the same concept later to establish the expression for induced emf. The usual applications of electromagnetism are augmented by a description of the cyclotron, e/m measurements, the mass spectrograph, and the betatron. The magnetic properties of matter are developed on the basis of Ampèrian currents and a description of the domain theory is included. In general the development of these sections concurs with the recently published recommendations of the A.A.P.T. Committee on Coulomb's Law.

The section on optics opens with a good survey of the nature and physical properties of light. A more explicit derivation of the thin lens relation in the paraxial case would be desirable but the text does use both the Gaussian and Newtonian form and contains a good section on optical instruments. The chapter on photometry is tied in with a chapter on colorimetry—a subject usually not discussed in other elementary texts. Physical optics is set out mainly as descriptive material and is replete with excellent photographs, but an analysis of the single slit diffraction pattern is wanting.

The two closing chapters of the book entitled Atomic Physics deal with spectra and natural and artificial radioactivity. However, considerable material which is usually appended in such a section has been well integrated into other parts of the book.

From the point of view of teachability this textbook ranks high. The main stress is on fundamental physical ideas. Examples are drawn more freely from physical situations than from more specific engineering cases. The authors are careful in defining terms and consistent in developing ideas and principles.

The few numerical errors in physical constants that occur can easily be spotted. One would expect a more analytic treatment of interference and diffraction and at least a mention of surface phenomena in a text of this type. The reviewer considers the absence of a discussion of kinetic theory an even more serious shortcoming in the otherwise excellent section on heat.

The text as mentioned stems from the *Principles of Physics*. A companion volume entitled *College Physics* by Sears and Zemansky differs from the *University Physics*

only in the degree of mathematical treatment, the *College Physics* using no calculus. Each of the three contains an ample number of problems, some of which require an analysis that really tests the students' comprehension of the subject. Of these three offerings in textbooks at least one should appear as custom-built for the mental measurements of the student.

I. WALERSTEIN
Purdue University

Physical Constants. Fifth edition. W. H. J. CHILDS. Pp. 77, $4\frac{1}{4}$ in. $\times 6\frac{3}{4}$ in. John Wiley and Sons, Inc., New York, and Methuen and Company, Ltd., London, 1949. Price $1.25.

This very little book is one of a series entitled "Monographs on Physical Subjects." It is not intended to replace the larger reference books of physical constants and mathematical tables, but has extracted from these more detailed tabulations the information which is needed most frequently, and gathered it into a booklet so small and light that it can easily be carried in the pocket of student or instructor.

The mathematical tables are restricted to four-figure logarithms and antilogarithms, natural sines, cosines, tangents, reciprocals, and squares. Some information, such as that needed for the corrections to a barometer reading, for finding the electric resistance of wires, and for converting from electron volts of quantum energy to wavelength and and frequency, is presented in the form of convenient and easily read nomograms. The remainder of the space is occupied by such obvious needs as tables of densities, thermal constants, wavelength data, radioactivity constants, etc. Everything that has been included appears to be worth the space it occupies; to have included more in an attempt to satisfy all needs would have defeated the purpose of the book. The cgs system of units is used exclusively: a future edition might at least recognize the existence of the mks system.

REGINALD T. HARLING
St. Lawrence University

Cosmological Theory. Second edition. G. C. McVITTIE. Pp. 103, $4\frac{1}{4} \times 6\frac{3}{4}$ in. Methuen and Company, Ltd., London, 1949. Distributed in the United States by John Wiley and Sons, Inc., New York. Price $1.50.

This contribution to the Methuen series was first published in 1937. The author, pointing out that a major revision could not profitably be made until new data gained with the Mt. Palomar telescope are available, has made minor changes only in the present edition.

It might be said that Professor McVittie's book (to compare it with three of the "classics" of contemporary cosmological theory) has not the explicit and detailed clarity of Tolman's *Relativity, Thermodynamics, and Cosmology*, the deft coherence of Robertson's 1933 *Reviews of Modern Physics* monograph, nor the speculative charm of Eddington's *The Expanding Universe*. Nonetheless, it is a useful and important book. In the first chapter the salient

astronomical observations for cosmology are presented. The elements of tensor calculus and of general relativity are compactly presented in two succeeding chapters, and then utilized in the final two chapters on the "Expanding Universe" and Milne's "Kinematical Theory." The emphasis of the book is on explanation and correlation of fact in terms of theory, rather than on the construction of theory as such.

One achievement in general relativity theory since the first appearance of this book, which might well have been included in the present edition, is the demonstration that the laws of motion of material particles are a necessary consequence of the general inertial field equations. Also, no mention is made of the possibility of omitting the cosmological constant from the field equations—a possibility strongly favored by Einstein because of the *ad hoc* grounds on which he originally introduced the constant.

In his discussion of general relativity theory, as well as in his inclusion of a chapter on kinematic relativity, the author shows the influence of E. A. Milne on his thought. This aspect of the book adds to its completeness as a survey of cosmological theory, and perhaps makes the work particularly useful in this country where Milne's work has been little discussed. The author criticizes general relativity theory on the grounds that the various space-time coordinate systems that are possible, under the covariance principle of the theory, lack physical meaning or specification. In discussing Milne's theory the author likewise stays within the respected philosophic tradition of Scotch empiricism. The kinematic theory is presented not so much in terms of its development as a logical system on an epistemological postulate, as has been stressed by Milne himself, but rather as a system whose defined coordinates can be readily applied in a theory of the observed cosmological facts. Some objections to Milne's introduction of two different time scales are suggested.

RICHARD SCHLEGEL
Michigan State College

The Metre-Kilogram-Second System of Electrical Units. R. K. SAS AND F. B. PIDDUCK. Pp. 60+v, $4\frac{1}{4} \times 6\frac{3}{4}$ in. Methuen and Company, Ltd., London, 1947. Distributed in the United States by John Wiley and Sons, Inc., New York. Price $1.00.

The first five chapters are introductory and point out, rather humorously, the disadvantages of the old systems of units. Chapters 6 and 7 define the mks units of mechanics and general physics; Chapters 8–13 develop, with numerous examples, the mks system for electricity and magnetism; and in Chapters 14–16 some of the equations of modern physics, including the Schroedinger equation, are expressed in the new units. Chapters 17–19 give a summary and list of formulas. The mks system used throughout the book is the rationalized system.

All units are defined in terms of other mks units; and there are no conversion tables from the older systems because such tables "would only encourage the reader in the belief that the new system is subsidiary." The reader who has been brought up on the older systems will probably

feel, because of this omission, as if he were trying to learn a foreign language without being allowed to use a dictionary. It may, however, be a useful exercise for the reader to work out the conversion formulas for himself.

Two or three minor flaws in the book should be mentioned. The word "oersted" is used to mean the mks unit that other authors call the "ampere-turn per meter." The letter J, set in three different styles of type, has been used to represent three different and unrelated quantities. The subject of electricity and magnetism is introduced by magnetostatics, in which mks units show up rather badly.

These small defects do not really detract from the quality of the book; and anyone who studies through it will be rewarded by having a thorough knowledge of the new units when he finishes.

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Dipole Moments. Second edition. R. J W. LE FÈVRE. Pp. 117+v, $4\frac{1}{2}\times6\frac{3}{4}$ in. Methuen and Company, Ltd., London, 1948. Distributed in the United States by John Wiley and Sons, Inc., New York. Price $1.25.

This book gives an account of the measurement of the permanent and induced dipole moments of molecules, and of the applications of the results in the investigation of molecular structure. It is written from an experimental rather than a theoretical point of view. The first chapter deals with the theories of molecular dipole moments and develops formulas for computing them from measurements of dielectric constants and refractive indices. Then follows a description of the experimental procedures used to determine molecular dipole moments of compounds in gaseous form and of compounds in solution. In Chapter 4 the author discusses the determination of the dipole moment of a specific link in a molecule by comparison of the measured moments of different molecules, with application to the problem of determining the arrangement of the atoms in the molecule. The other chapters deal with corrections for various effects, particularly solvent effects when solutions are used, and molecular rotation. At the end of the text is a table of values of dipole moments of various molecules, mostly organic.

The first edition of the book was published in 1938, and this second edition differs from it only in a few details. In particular, the present edition contains hardly any references to work done since 1938, although it is generously supplied with references to earlier work.

The author is a chemist, and accordingly some knowledge of chemistry is assumed on the part of the reader. Theoretical details have, however, been held to a minimum; no quantum theory is used; and the derivations of formulas are, for the most part, based on simple molecular models.

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LETTERS TO THE EDITOR

Some Observations on Chladni Figures

THE demonstration of Chladni figures with round and square plates is usually done with either fine sand or lycopodium powder. I wish to relate here a very interesting observation which appears not to have been reported and which incites some critical thinking on the part of the student and, indeed, on the part of the teacher.

My supply of both sand and lycopodium powder being very low, I decided to mix what I had of each. This mixture I spread on a plate clamped at the center. The plate was excited by bowing (quite vigorously) and then constrained at one point by the touch of a finger. The results were very definitely unexpected. With sand alone or with lycopodium alone it is observed that each accumulates along the nodal lines, and these are governed both by the bowing and the damping, as well as by the properties of the plate. In the case I report, however, the sand did mass itself along the nodal lines (nodal circles and radii), but the much lighter lycopodium powder collected more prominently in the middle of the vibrating segments, and not along the nodal lines! This leads to some interesting inquiry and I here pose the questions: Why does it do this? What mechanism prevails? It might be added that this effect was more pronounced under very vigorous bowing than under a gentle bowing, and, of course, it is necessary to bow more vigorously for sand than for the lighter lycopodium powder.

JULIUS SUMNER MILLER

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A Simple Experiment on Heat

THIS experiment requires apparatus consisting of two ring stands, two bunsen burners, safety matches, a sheet of hard asbestos board and a sheet of aluminum of approximately the same dimensions (6 in. long by $3\frac{1}{2}$ in. wide by $\frac{1}{8}$ in. thick). The ring stands are set side-by-side with the sheet of aluminum on one and the sheet of asbestos on the other. Two matches are placed on each sheet; one in the middle directly above the bunsen burner and one on the edge, two or three inches away. In what order will the matches catch fire after the burners are lighted?

ROBERT KATZ

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Old Problems Stated Anew

1. If a vessel of liquid containing an undissolved solid is stirred with a rotary motion and the stirrer withdrawn the solid collects in the *middle* of the vessel and not around the peripheral edges. If, however, the vessel is rotated on a rotating platform the solid does go to the *outer edges* of the vessel. The second case is easily explained. What shall we say about the first case?

2. Why does frozen, powdery snow squeak when stepped on?

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